Physics Notes

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WARNING: NOT PROOF READ YET
This compilation of notes are to be used as a reference for the GCE "A"-level Physics paper, as a refresher in definitions, theories as well as for general descriptions of presentation form. These notes are meant for free, public use, but at the reader's own risk.
Good luck with your exams.

1 Measurements

1.1 Units

Physics can be summarized as a collection of mathematical relationships between physical phenomena. Each and every physical quantity has a numerical magnitude and a unit. Note that it is nonsensical to compare a physical quantity to a unit (e.g. time cannot be compared to seconds).

$$\underbrace{F}_{\text{Physical Quantity}} = \underbrace{5}_{\text{Numerical Magnitude}} \underbrace{N}_{\text{Unit}}$$

Definition 1.1: SI Base Units

SI base units are a selection of fundamental physical quantities, from which all other physical quantities can be represented as a combination of SI Base Units. These quantities have been arbitrarily chosen for accessibility and reproducibility.

Definition 1.2: Derived Units

Derived Units are defined as products or quotients of base units and are obtained as products of base units

Base Quantity	Base Unit	Symbol
Time	Second	s
Length	Meter	m
Mass	Kilogram	kg
Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol

For a mathematical operation to be valid, addition and subtraction between physical quantities have to have the same unit and two sides of an equation must have the same unit.

Definition 1.3: Homogeneous Equations

An equation is homogeneous if both sides of an equation have the same resultant units. Also called Dimensionally Consistent.

The homogeneity of an equation can be used to determine the powers of physical quantities used to derive a value.

1.2 Numerical Magnitudes

Orders of magnitudes of a physical quantity can be used to represent decimal multiples of a number.

Prefix	Symbol	Power of 10
tera	T	12
giga	G	9
mega	M	6
kilo	k	3
deci	d	-1
centi	c	-2
milli	m	-3
micro	μ	-6
nano	n	-9
pico	p	-12

Definition 1.4: Standard Form

Standard form is where the numerical magnitude of a physical quantity is written in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Estimation of the order of magnitude of a physical quantity can be derived from estimating component values of a certain order of magnitude and then applying physical equations.

1.3 Error

Error in a reading is where there is uncertainty in the exact value of the numerical magnitude of a physical quantity.

Definition 1.5: Systematic Error

Systematic errors are caused by lapses in the measurement process, resulting in values consistently erroneous to give always smaller or always larger readings and can be eliminated if the source of error is known and accounted for.

Definition 1.6: Random Error

Random errors are caused by inherent inaccuracy and lack of precision in a reading, resulting in values scattered about a mean and can be mitigated by repeating measurements and finding lines of best fit but otherwise cannot be predicted.

Definition 1.7: Accuracy

Accurate readings are values which are close to the true value of a physical quantity and is influenced by systematic error.

Definition 1.8: Precision

Precise readings are values which agree which other and is influenced by random error.

1.3.1 Measuring Values

Precision of a measuring instrument is determined by its least count. Measurements of length and volume are read to their least count, or half their least count if the markings are larger than 1mm such as on a meter rule or a graph. Digital instruments are read and recorded to their displayed value except for tools which depend on other erroneous input such as human reaction time. Do note that the ruler is a special case, where since the error in reading is 0.5mm but two readings are made (one for the starting point of measurement, and one for the ending point, te result is obtained by subtracting starting value from ending value, though starting is usually at the zero mark) the total error is twice that error or 1mm. In questions which specify that the error accompanying each reading is one division, the absolute error is twice the least count.

1.3.2 Error Propagation

Equation 1.1: Error Propagation

For a resultant value Q, two derivative values X and Y and their powers or coefficients a and b

$$Q = aX + bY \quad \Delta Q = |a|\Delta X + |b|\Delta Y$$
$$Q = kX^aY^b \quad \frac{\Delta Q}{Q} = |a|\frac{\Delta X}{X} + |b|\frac{\Delta Y}{Y}$$

Absolute uncertainty is represented to 1 s.f. while fractional and percentage (fractional multipied by 100%) uncertainty is represented to 2 s.f. .

To find the situation where maximum fractional error occurs, adjust the values such that the value of Q is its smallest possible value.

1.3.3 DP and SF

Addition and subtraction operations in experimental situations require the result to follow the largest decimal place value of its derivatives. Multiplication and division operations in experimental situations require the result to follow the least significant figures of its derivatives. However, in exam settings seek to maintain all working in 5sf/dp and only reduce sf/dp when obtaining answers.

1.3.4 Scalars and Vectors

Definition 1.9: Scalar Quantity

A Scalar Quantity is a physical value with a numerical magnitude, and are represented by a magnitude and a unit.

Definition 1.10: Vector Quantity

A Vector Quantity is a physical value with a numerical magnitude as well as a direction, and are represented by a magnitude, unit and a direction.

Before solving questions involving vector quantities, a positive direction should be defined as whichever direction is most convenient.

2 Kinematics

Definition 2.1: Distance

Distance x is the length of a path followed by an object, measured in m.

Definition 2.2: Displacement

Displacement s is the distance moved in a specified direction from a reference point, measured in m. It is the vector equivalent of distance.

Definition 2.3: Speed

Speed v is the instantaneous speed of an object, defined as the rate of change of distance traveled with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average speed refers to the distance traveled over a significantly large time taken.

Definition 2.4: Velocity

Velocity v is the instantaneous velocity of an object, defined as the rate of change of displacement with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average velocity refers to the change in displacement over a significantly large time taken. It is the vector equivalent of speed.

Definition 2.5: Acceleration

Acceleration a is the instantaneous change in velocity of an object, defined as the rate of change of velocity with respect to time, measured in $\mathrm{m\,s^{-1}}$. Average acceleration refers to the change in velocity over a significantly large time taken.

Note that when faced with a kinematics graph (s/v/a against t), the gradient of the graph (differential) and the area under the graph (integral) obtain special meanings.

2.1 Equations of Motion

For a situation involving uniform acceleration and motion in a straight line, the following equations hold:

Final velocity from initial velocity and acceleration

$$v = u + at$$

Displacement from average velocity

$$s = \frac{1}{2}(u + v)t$$

Displacement from initial velocity and acceleration

$$s = ut + \frac{1}{2}at^2$$

Final velocity from displacement,
initial velocity and acceleration

$$v^2 = u^2 + 2as$$

For the condition of objects in freefall, acceleration is equal to which takes the value of 9.81 $\mathrm{m\,s^{-1}}$.For the conditions of objects in projectile motion with the assumption of no air resistance, acceleration in the vertical dimension behaves as if the object is in freefall, and acceleration in the horizontal direction is equal to zero.

2.2 Air Resistance

When objects move through air, it experiences viscous drag or air resistance. Air resistance acts opposite to the direction of velocity and is proportional to the velocity, or at higher velocities is proportional to the square of the velocity. The terminal velocity is the velocity at which the air resistance is equal to accelerative forces on an object, hence the resultant acceleration is equal to zero.

For an object projected upwards in freefall, the time of flight upwards will be smaller than the flight downwards.

On the way up, air resistance acts against upward motion and hence acts downwards in line with gravity, creating a larger resultant force downwards and a larger acceleration which retards its vertical motion, hence the velocity decreases at a faster rate and it takes less time to travel to the peak of the trajectory than if there had been no air resistance.

On the way down, air resistance acts against downward motion and hence acts upward and against gravity, reducing the resultant force downwards and a lower acceleration which accelerates the object downward, hence the velocity increases at a slower rate and it takes more time to travel the same distance downward had there been no air resistance.

3 Dynamics

Definition 3.1: Force

Force F is an action that causes a change in the physical shape or state of a body and is defined as the product of mass and acceleration with units N or $kg\,m\,s^{-1}$

Multiple forces acting upon a body can be added together in a vector sum to find the resultant force.

3.1 Newton's Laws of Motion

Definition 3.2: Newton's First Law of Motion

Newton's First Law of Motion states that a body continues in its state of rest or motion in a straight line unless acted upon by an external force.

Definition 3.3: Newton's Second Law of Motion

Newton's Second Law of Motion states that the change in momentum of a body is proportional to the resultant force acting on it and occurs in the direction of said resultant force.

Definition 3.4: Newton's Third Law of Motion

Newton's Third Law of Motion states that for a force acting from a first body on a second body, there is an equal and opposite force acting from the second body on the first body.

3.2 Equilibrium

Definition 3.5: Inertia

Inertia is the tendency of a body to maintain its current motion or lack thereof unless acted upon a force.

Definition 3.6: Equilibrium

When a body experiences forces which do not change its state

For a object to be in equilibrium, the resultant force on the object must have zero magnitude and the resultant torque on the object about any axis must also have zero magnitude.

3.3 Momentum

Definition 3.7: Momentum

Momentum is defined as the product of the mass of an object and its velocity with units $kg\,m\,s^{-1}$.

The total momentum of a system is equivalent to the vector sum of its component objects' momenta. Forces can be simplified into the change of momentum over time.

3.4 Action Reaction Pairs

Definition 3.8: Action Reaction Pairs

Action Reaction Paris are pairs of forces which arise due to Newton's Third Law of Motion which then are of the same type (Normal Contact with Normal Contact, Friction with Friction, Electric with Electric) and act upon different bodies, in addition to properties described in the law which states that the forces are equal in magnitude but opposite in direction.

3.5 Impulse

Definition 3.9: Impulse

Impulse is defined as the product of a force acting on an object and the time which the force is exerted, alternatively the amount of momentum that is transfered, with units $N\,s$ or $kg\,m\,s^{-1}$.

Change in momentum can also be measured by finding the area under a Force-Time graph.

3.6 Drawing Forces

Free body diagrams are rudimentary drawings which illustrate the location, direction and magnitude of multiple forces acting upon objects. When drawing diagrams, label with full names of forces unless the short forms are already defined (or define them yourself in a section of the question paper).

Weight W is drawn from the center of mass downward.

Normal Contact Force N is drawn from the point of contact between two bodies. For two contacting surfaces, the force is drawn perpendicular to the surface.

Frictional Force f is drawn on the surface which friction acts on.

Tension T / Compression is drawn along the wire, spring or strut. Tension is drawn inward while compression is drawn outward.

Upthrust U is drawn upwards from the center of mass which is below water level.

Viscous Force F_v is drawn from the center of the surface furthest from the direction of motion and is opposite to the direction of motion.

Lift L is drawn perpendicular to the axis of wings.

Resultant F_{net} is drawn disconnected from the body and is drawn with two arrows in the direction of motion.

3.7 Collisions

Extremely quick collisions between objects involve high values of F such that assessment is more feasible by examining changes in momentum rather than forces exerted.

Definition 3.10: Law of Conservation of Momentum

The Law of Conservation of Momentum states that the total momentum of a system remains the same when no external force is applied.

Conservation of Momentum provides the equation:

Equation 3.1: Conservation of Momentum

For mass m and initial and final velocities u and v

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Definition 3.11: Law of Conservation of Energy

The Law of Conservation of Energy states that energy can neither be created nor destroyed, hence the total energy of a closed system remains the same.

Conservation of Energy provides the equation:

Equation 3.2: Conservation of Kinetic Energy

For mass m and initial and final velocities u and v

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

Definition 3.12: Elastic

Elastic collisions maintain the property of conservation of momentum as well as conservation of kinetic energy.

Combining the equations of conservation of momentum and conservation of energy, we obtain this in the case of an elastic collision:

Equation 3.3: Elastic Collision: Constant Relative Speed

For initial and final velocities u and v

$$u_1 - u_2 = v_2 - v_1$$

Definition 3.13: Completely Inelastic

Completely Inelastic collisions maintain the property of conservation of momentum but involve the conversion of kinetic energy to other forms of energy. Particles stick to each other after collision.

4 Forces

4.1 Elastic Force

Definition 4.1: Hooke's Law

Hooke's Law states that the extension of a spring is proportional to the applied force if the limit of proportionality is not exceeded.

Equation 4.1: Elastic Force

For some distance of extension x, some proportionality constant k and some force F, the values are related by the equation

$$F = kx$$

Equation 4.2: Elastic Energy

For some distance of extension x and some proportionality constant k, the energy stored in a spring E is given by the equation

$$E = \frac{1}{2}kx^2$$

4.2 Frictional Force

Definition 4.2: Friction

Friction is a force which exists between two surfaces in contact with each other and resists motion between these two surfaces.

Friction is drawn along the line of contact between two objects. Note that friction between a wheel and a surface is in the direction of motion.

Equation 4.3: Frictional Force

For a given normal contact force N between two surfaces with a frictional constant μ the frictional force f is given by the equation

$$f = \mu N$$

4.3 Upthrust

Definition 4.3: Fluid

A Fluid is a substance which can flow, including most liquids and gases.

Definition 4.4: Density

Density ρ of a substance is its mass per unit volume, with units $\mathrm{kg\,m^{-3}}$

Definition 4.5: Pressure

Pressure p is the force per unit area exerted at right angles to a surface by some object, with units $\mathrm{N\,m^{-1}}$ or $\mathrm{kg\,m^{-1}\,s^{-1}}$

Note that when considering pressure in a liquid at sea level, the pressure due to atmosphere needs to be accounted for.

Equation 4.4: Pressure in Fluid

For the height $h$of a fluid above the level considered, its density ρ and the gravitational acceleration g at a point, pressure p is given by the equation

$$p = h\rho g$$

Definition 4.6: Upthrust

Upthrust is a vertically upward force exerted on a body by a fluid when it is fully or partially submerged in a fluid due to difference in fluid pressure at different heights.

Definition 4.7: Archimedes Principle

The Archimedes Principle states that the upthrust on a submerged object is equal to the weight of liquid displaced by said object.

Questions on upthrust usually involve a calculation of density, mass or volume of liquids or solids in a system. In the case of floating objects, utilize the equation $U = W$ to find the weight or upthrust experienced by an object.

4.4 Viscous Force

Definition 4.8: Viscous Force

Viscous Force is the force experienced by a body moving through a fluid when it receives normal contact force from the particles of the fluid after it imparts momentum onto fluid, written F_v.

The magnitude of the viscous force depends on the shape of the body and viscosity of the fluid, as well as the speed of the body which has a proportional relationship to the force at low velocity and a squared relationship with the force at higher velocities.

F_v is zero when a body is at rest. When a body affected by viscous force experiences a constant force or acceleration, the body speeds up at a decreasing rate as the resultant force is smaller due to the increasing viscous force. Terminal velocity is reached when the viscous force is equal to the applied force, resulting in equilibrium.

Definition 4.9: Terminal Velocity

Terminal Velocity is the speed at which the viscous force experienced by a body prevents further acceleration.

4.5 Calculating Equilibrium

A body in equilibrium must have both translational and rotational equilibrium. As such, it needs to have zero resultant force as well as zero torque about any axis.

Translational equilibrium is obtained when all forces acting on a body are added using vector addition and have zero resultant magnitude. Forces can be resolved into their dimensional components and summed together.

Definition 4.10: Principle of Moments

For any body in rotational equilibrium, the sum of all clockwise moments about an axis is equal to the sum of all anticlockwise moments.

Definition 4.11: Moment

A Moment is a physical value which involves the multiplication of a perpendicular distance from an arbitrary axis with another physical quantity existent at a point.

Definition 4.12: Torque

The Torque of a force about an arbitrary axis is defined as the product of the force and the perpendicular distance from the point to the line of action of the force, with units N m or kg m s^{-2}.

Definition 4.13: Couple

A Couple is a pair of equal and opposite parallel forces whose lines of action do not meet.

Equation 4.5: Torque of A Couple

For the magnitude of one force in the couple F and the perpendicular distance between the two forces d, total torque τ is given by the equation

$$\tau = Fd$$

A couple has the special quality that it has zero resultant force but still has a torque. A couple will continue to rotate until the lines of action of the two forces coincide and have zero perpendicular distance. Multiple forces which all pass through one single point have no net torque.

4.6 Calculating Center of Mass

Definition 4.14: Center of Gravity

The Center of Gravity of an object is the point where gravitational attraction on the body appears to act.

CG is calculated by finding the point where when used as a pivot results in rotational equilibrium. This means that the point is vertically in line with the center of gravity. CG of an irregularly shaped object can be obtained by pivoting the body and drawing a line vertically down from the pivot multiple times, where the point where the lines intersect would be the center of gravity.

5 Work Energy and Power

5.1 Work

Definition 5.1: Work

Work WD is defined as the product of a force and the displacement in the direction of the force, measured in Joules J or kg m s^{-2}

Negative work done is a sign of a dissipative force.

Equation 5.1: Work Done on a System

For a force F, displacement s and angle between Force and Displacement θ, work done W is given by the equation

$$W = Fs\cos(\theta)$$

Equation 5.2: Work Done by a Gas

For a contained gas of changing volume V and external pressure p, work done $W$is given by the equation

$$W = p\Delta V$$

Amount of work done can be measured as the integral of the Force-Distance graph for normal motion, the integral of the Pressure-Volume graph for work done by a gas and the integral of the Force-Extension graph for work done by a spring.

5.2 Energy

Definition 5.2: Energy

Energy is the quantification of an object's capacity to do work,measured in J or kg m s^{-2}

Equation 5.3: Kinetic Energy

For a object of mass m and speed v, the amount of Kinetic Energy E_k contained is given by the equation

$$E_k = \frac{1}{2}mv^2$$

Note that an object whose velocity changes has an energy change of $\frac{1}{2}m(v^2 - u^2)$ rather than $\frac{1}{2}m(v-u)^2$.

Equation 5.4: Gravitational Potential Energy

For a object of mass m, gravitational acceleration g and (relative) height h, the amount of Gravitational Potential Energy E_p contained is given by the equation

$$E_p = mgh$$

Equation 5.5: Elastic Potential Energy

For a spring of proportionality constant k and extension x, the amount of Elastic Potential Energy U_E contained is given by the equation

$$U_E = \frac{1}{2}kx^2$$

Given the Energy-Distance graph of a object experiencing a field force, the gradient of the graph gives the force at a certain distance.

Definition 5.3: Principle of Conservation of Energy

The Principle of Conservation of Energy states that energy cannot be destroyed or created, only converted and transferred.

The sum of all kinetic and potential energy at any point in time is constant, even in the presence of a dissipative force so long as no work is done on a system. Dissipative forces are also lessened in a system with more uniform motion.

Equation 5.6: Efficiency

To obtain the efficiency η of a system, use the equation

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Definition 5.4: Power

Power is the quantification of work done with respect to time, defined as the rate which work is done with respect to time or the amount of energy transferred with respect to time, measured in W or kg m s^{-2}

For questions involving mass flow rates, calculate values in terms of their algebraic quantities and then cancel out the time quantity at the end to find the rate.

6 Circular Motion

6.1 Kinematics of Circular Motion

Definition 6.1: Angular Displacement

Angular Displacement θ is the angle an object makes with reference to a line, measured in radians.

Definition 6.2: Radian

A Radian is the angle subtended by an arc of length equal to its radius.

Definition 6.3: Angular Velocity

Angular Velocity ω is defined as the rate of change of angular displacement with respect to time, measured in rad s^{-1}

Definition 6.4: Period

A period of a system T is the time taken for a system to complete one cycle of motion, measured in s

Period, with regard to circular motion, is the time taken for one complete revolution to finish.

Equation 6.1: Linear Velocity

For some angular velocity ω and some radius r, the linear velocity of an object in uniform circular motion is given by the equation

$$v = r\omega$$

6.2 Dynamics of Circular Motion

A object in uniform circular motion orbits an object at a constant radius, linear velocity and angular velocity, but with a changing direction.

Velocity is constantly changing since direction is changing despite linear velocity remaining the same, hence there is a force acting on the body.

Linear velocity is constant, hence the force acting on the body is perpendicular to the direction of motion in order to keep the linear velocity unchanged. As such, no work is done on the force as well.

Equation 6.2: Centripetal Acceleration

For a radius r, linear velocity v and angular velocity ω, the centripetal acceleration a is given by the equation

$$a = r\omega^2 = v\omega = \frac{v^2}{r}$$

Definition 6.5: Centripetal Force

Centripetal Force F_c is a name given to any force which allows a body to undergo circular motion, typically calculated as the product of centripetal acceleration and the mass of an object, measured in N.

6.3 Special Cases

6.3.1 Racecar on Inclined Track

Horizontal component of normal force of ground on the car provides centripetal force. When the car goes above or below the speed needed to maintain this centripetal force, friction between the car tires and the track contributes additional force towards maintaining circular motion. Alternatively, the car will experience horizontal acceleration in the event it is no longer able to maintain circular motion and "slide" up and down the incline.

6.3.2 Vertical Circular Motion

Apparent weight is given by the normal contact force acting on an object. Since centripetal force is the vector sum of gravity and normal force, at the bottom of a loop where centripetal force acts upwards but gravity acts downwards, the normal contact force is largest so as to act against gravity to obtain the necessary centripetal force, hence apparent weight is highest at the bottom.
Objects falling at the top of circular motion / objects (not) in contact with other objects can be explained by saying that "contact" is caused by having a normal force between two objects, and at the top of a loop and at low linear velocity the weight of an object is higher or lower than its necessary centripetal force, hence there is (no) normal force between object and another object.

Alternatively, argue that objects in circular motion tend to move tangentially to the path of circular motion and at sufficient speeds press against other objects rather than fall due to gravity.

6.3.3 Vertical Circular Motion with Uniform Speed

Note that due to the presence of gravitational acceleration, any additional force required to act upon an object to keep it in circular motion changes at different stages of the rotation in order to keep total centripetal acceleration constant.

Also note that work is done in the process of maintaining uniform speed as E_k is constant but E_p changes.

6.3.4 Vertical Circular Motion with Non-uniform Speed

In order to maintain circular motion throughout a vertical loop without any external work being done, the object must have enough energy at the bottom of the loop such that it can reach the top of the loop and still have the necessary linear velocity given the centripetal acceleration provided by gravity. Velocity at the bottom is typically in the form $v_{\text{bottom}} = \sqrt{5gr}$ while velocity at the top is in the form $v_{\text{top}} = \sqrt{gr}$, but its derivation is required to be shown.

7 Gravitation

Definition 7.1: Gravitation

Gravitation is defined as the attractive force between two masses.

Equation 7.1: Gravitational Equations

For the mass of a planetary object m_1 , the radius from it r, and the gravitational constant G, the force experienced F by a secondary mass m_2 , the strength of gravitational field or acceleration due to gravity g, the potential energy U of a secondary mass m_2 and the potential due to the planet ϕ are given by the equations

$$F = -G\frac{m_1 m_2}{r^2}$$
$$g = -G\frac{m_1}{r^2}$$
$$U = -G\frac{m_1 m_2}{r}$$
$$\phi = -G\frac{m_1}{r}$$

7.1 Gravitational Force

Definition 7.2: Newton's Law of Gravitation

Newton's Law of Gravitation states that every point mass attracts every single other point mass along a line intersecting both points, which is proportional to the product of the two masses and inversely proportional to the square of the distance between the two masses.

Gravitational force is a field force, which does not require contact between two objects to have effect. Gravitational force is also a case of an inverse-square law.

Gravitational Force is the basis which other gravitational quantities are calculated on.

7.2 Gravitational Field

Gravitational Fields are drawn as a set of field lines which demonstrate the direction and magnitude of acceleration at a certain point. Lines are drawn pointing towards masses and more dense field lines indicate stronger fields.

Definition 7.3: Gravitational Field Strength

The Gravitational Field Strength $g$at a point is the gravitational force experienced per unit mass at a point, with units $\mathrm{m\,s^{-1}}$.

Gravitational field varies in the case of hollow or solid masses as well as whether the object is inside or outside the mass. Hollow masses have zero gravitational field inside a mass. Solid masses have fields which vary linearly inside of them and fields which follow inverse square law outside of them.

When calculating gravitational field strength which involves some element of circular motion (planetary rotation, object in orbit), do consider that observed acceleration is a composite of acceleration due to gravity and centripetal acceleration.

7.3 Gravitational Potential

Definition 7.4: Gravitational Potential Energy

The Gravitational Potential Energy U of a mass at a point is the work done by an external force in bringing a test mass from infinity to that point, with units J.

Negative total energy indicates that a mass is bounded to the gravitational field of said mass.

Definition 7.5: Gravitational Potential

Gravitational Potential ϕ at a point is the work done per unit mass by a external force in bringing a test mass from infinity to a point, with units $\mathrm{J\,kg^{-1}}$.

Gravitational Potential (Energy) is negative because the direction of displacement is opposite that of the external force.

7.4 Escape Velocity

The escape velocity of a body is the velocity required such that said body is able to travel to infinity away from a planetary mass. Escape velocity is independent of the mass of the escaping object. Escape from Earth requires a speed of about $11.2 \times 10^3\,\mathrm{m\,s^{-1}}$.

Equation 7.2: Escape Velocity

For gravitational acceleration g and radius from the original point R, the escape velocity v is given by the equation

$$v = \sqrt{2gR}$$

7.5 Planetary Orbit

Definition 7.6: Kepler's Third Law

Kepler's Third Law states that the square of the period of a planetary orbit is proportional to the cube of its radius. Mathematically, it is presented as

$$T^2 \propto R^3$$

Given the radius and period of one planet's orbit, you can then infer mathematically the period or radius of another planet when given one of the two variables.

7.5.1 Geostationary Orbit

A geostationary satellite maintains the same position relative to a point on a planet's surface.

- Orbital period is same as the planet's rotation period (24 hours for Earth)
- Plane of orbit is the same as the planet's equator
- Direction of orbit is same as the planet's direction of rotation (eastward for Earth)

Geostationary orbits allow for uninterrupted surveillance of one point on a planet and is easier to communicate with, and has a high field of view due to its large orbital radius. However, geostationary orbits face a significant loss in signal strength due to the large radius, as well as poorer quality in terms of imaging satellites as well as significant latency in signal.

8 Thermodynamics

8.1 Temperature and Heat

Definition 8.1: Internal Kinetic Energy

Internal Kinetic Energy E_K is the amount of energy stored in an object's molecule's translational and rotational energy.

Definition 8.2: Temperature

Temperature is a quantization of the internal kinetic energy of an object.

Definition 8.3: Heat

Heat is the flow of internal kinetic energy due to the difference in temperature between them.

Definition 8.4: Thermal Equilibrium

Objects that are in Thermal Equilibrium have no net flow of heat, and occurs if and only if these objects have the same temperature.

Definition 8.5: 0^{th} Law of Thermodynamics

The 0^{th} Law of Thermodynamics states that if objects A and B are in thermal equilibrium and B and C are in thermal equilibrium, then objects A and C are in thermal equilibrium.

8.2 Temperature Scale

Equation 8.1: Thermodynamic Temperature

For an object with temperature T_C in °C, its thermodynamic temperature $T_K$in K is given by the equation:

$$T_K = T_C + 273.15$$

Definition 8.6: Kelvin

The Kelvin K is a measurement of temperature. 1 K is $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water (0.01 °C).

8.3 Ideal Gas Law

Three equations which relate various quantities of a gas, namely Boyles Law $p \propto \frac{1}{V}$, Charles' Law $V \propto T$ and The Pressure Law $p \propto T$, an equation relating these three values and the amount of gas is derived.

Equation 8.2: Ideal Gas Law

For an ideal gas with pressure p, volume V, amount of gas in mol n and thermodynamic temperature T

$$pV = nRT$$

Definition 8.7: Ideal Gas

An Ideal Gas satisifes the relationship of $pV = nRT$.

An Ideal Gas is assumed to have the following properties for ease of calculations:

1. Negligible intermolecular forces
2. Hard and identical particles with elastic collisions
3. Particles have negligible volume
4. Molecules move randomly

8.4 Internal Kinetic Energy of a Gas

Definition 8.8: Internal Energy

The Internal Energy of a gas is the sum of its microscopic kinetic energies (translational, rotational and vibrational) and potential energies of the system.

Equation 8.3: Internal Kinetic Energy

The average Internal Kinetic Energy $\langle E_K \rangle$ of a gas molecule with mass m and mean square speed c or thermodynamic temperature T using the Boltzmann constant k is given by the equation:

$$\langle E_K \rangle = \frac{1}{3} m \langle c^2 \rangle = \frac{3}{2} kT$$

The derivation of the expression for $\langle E_K \rangle$ is as follows:

1. Pressure in a container arises when gas molecules collide against a container.
2. Model the collision of gas molecules with container as a elastic collision, where a gas molecule has initial momentum mc. Due to conservation of kinetic energy, the final momentum of the molecule is $-mc$. The total change in momentum is $\Delta p = -2mc$.
3. Model the container as a cube of side length 2d. A collision occurs every time a molecule moves twice distance between two walls of the container, therefore it occurs once every $\Delta t = \frac{2d}{c}$.
4. Since force is defined as change of momentum per unit time, the force exerted of wall on the molecule is $F = \frac{\Delta p}{\Delta t} = -\frac{2mc^2}{2d} = -\frac{mc^2}{d}$.
5. By Newton's third law, the force of molecule on the wall is $F = \frac{mc^2}{d}$.
6. Considering the total force on the wall as a summation of the forces by each molecule, $F = \sum \frac{mc^2}{d} = \frac{Nm\langle c^2 \rangle}{d}$
7. Since pressure is force per unit area and the "unit area" is modeled as a square side of a container, $P = \frac{F}{d} = \frac{Nm\langle c^2 \rangle}{d^3} = \frac{Nm\langle c^2 \rangle}{V}$
8. Accounting for the fact that each of the three dimensions were arbitrarily chosen, pressure has to be divided by 3. $P = \frac{1}{3}\frac{Nm\langle c^2 \rangle}{V}$
9. Rearranging,

$$\frac{1}{3} Nm \langle c^2 \rangle = pV = NkT$$
$$m \langle c^2 \rangle = 3kT$$
$$\langle E_K \rangle = \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

Equation 8.4: Energy of Gaseous System

$$E_{\text{system}} = \frac{3}{2} NkT = \frac{3}{2} nRT = \frac{3}{2} pV$$

Equation 8.5: Energy of 1 Gas Particle

$$\langle E_K \rangle = \frac{3}{2} kT = \frac{1}{2} m \langle c^2 \rangle = E_{\text{system}} N^{-1}$$

Note that these equations only hold for a monatomic particle with no rotational or vibrational energy.

9 Thermodynamics

9.1 Heat Capacity

Definition 9.1: Heat Capacity

The Heat Capacity of a system is the ratio of energy input to temperature change of a system.

Equation 9.1: Heat Capacity

$$c = \frac{Q}{\Delta T}$$

Definition 9.2: Specific Heat Capacity

The Specific Heat Capacity of a material is the ratio of energy input to temperature change of a system per mass of that material.

Equation 9.2: Specific Heat Capacity

$$c = \frac{Q}{m\Delta T}$$

9.2 Kinetic Model of Matter

Matter primarily exists in three states: solid, liquid and gas. At ANY of these phases, the mean-square speed of these particles is still proportional to its temperature, but is NOT necessarily proportional to all of its internal energy. At the solid and liquid states, energy is stored within bonds between particles and is hence unable to be detected and measured in terms of temperature.

Phase changes occur when energy is released or used to form or break these bonds respectively. At these points, the temperature remains the same as energy is contributed to overcoming chemical bonds rather than increasing temperature of the object.

9.3 Latent Heat

Definition 9.3: Latent Heat of Fusion

The Latent Heat of Fusion is the amount of heat per unit mass required to cause a phase change from solid to liquid or vice versa.

Definition 9.4: Latent Heat of Fission

The Latent Heat of Fission is the amount of heat per unit mass required to cause a phase change from liquid to gas or vice versa.

The latent heat of fusion is less than the latent heat of fission because the phase change between solid and liquid only involves the breakdown of a solid lattice whereas the phase change between liquid and gas involves the breakdown of complete bonds between particles, in addition to the fact that energy needs to be supplied for a gas to form and do work against atmospheric pressure.

Evaporation is followed by a cooling effect because an evaporating particle may instantaneously have a above average amount of energy and be removed from the system, hence reducing the average energy of the remaining particles and leading to a drop in the average and hence a drop in temperature.

9.4 Calorimetry

In order to find the heat capacities of an object, static heating is used for solids whereas a method of continuous flow is used for fluids.

For solids, an object can be insulated from its surroundings and a immersion heater can be introduced to the object. The heat capacity is calculated from the energy dissipated from the heater and the change in reading of a temperature probe.

For fluids, the inability to insulate the system results in an alternative method which accounts for heat loss H. H is assumed to be constant when a system is at a constant temperature, hence a sample of the fluid is manipulated to flow through a pipe with varying mass flow rates and heat energy supplied to obtain the same temperature at the beginning and end of the pipe (typically using platinum resistance thermometers).

Equation 9.3: Specific Heat Capacity using Constant Flow

For two experimental setups x and x' with respective power supplied P and mass flow rate m; and identical time taken for sample t and terminal temperature change $T_{\text{out}} - T_{\text{in}}$, the specific heat capacity of the fluid is given by the equation

$$c = \frac{(P - P')t}{(m - m')(T_{\text{out}} - T_{\text{in}})}$$

9.5 Energy of a Thermodynamic System

Definition 9.5: First Law of Thermodynamics

The First Law of Thermodynamics states that as a result of conservation of energy, the change in energy of a gaseous system through a process is equal to the sum of the work done on the gas and the heat supplied to it and that Internal Energy is a state function.

Equation 9.4: First Law of Thermodynamics

For some work done on the gas WD and heat supplied to the gas ΔQ, the change in energy of a gaseous system ΔU is given by the equation

$$\Delta U = WD_{\text{on}} + \Delta Q$$

Definition 9.6: Internal Energy U

The Internal Energy U of a system is the total energy of a system, effectively the summation of microscopic kinetic energies of all molecules in the gas.

Equation 9.5: Internal Energy U

At a certain P and V, the internal energy of a system is given by the equation

$$U = \frac{3}{2}PV$$

Definition 9.7: Work Done WD

The Work Done WD of a system is the sum of energy used to compress or expand a system done by an external force.

Equation 9.6: Work Done WD

$$WD = -\int P dV$$
$$= -P\Delta V \quad \text{(at a constant } P\text{)}$$

The sign convention used by the A-levels is where the a positive WD indicates compression while a negative WD indicates expansion.

Definition 9.8: Heat Exchange Q

The Heat Exchange Q of a thermodynamic change is the amount of heat provided to a system.

9.6 $P - V$ Graphs

Definition 9.9: Thermodynamic Process

A Thermodynamic Process is a process which involves some change in the Pressure, Volume and Temperature quantities of a system. Processes are represented by arrows on a $P - V$ graph.

Definition 9.10: Cyclic Process

A Cyclic Process is a set of thermodynamic changes which start and end at the same point on a $P - V$ graph. This implies that across one cycle, the total change in internal energy is 0 and the net area under the cycle is numerically equal to ΔQ and $-WD$.

9.7 Thermodynamic Processes

Definition 9.11: Isobaric Process

An Isobaric Process is one where the pressure of a system remains constant. These processes are characterized by horizontal lines on a $P - V$ graph.

Equation 9.7: Isobaric Process

$$\Delta P = 0$$
$$WD = P\Delta V$$

Definition 9.12: Isothermal Process

An Isothermal Process is one where the temperature of a system remains constant. These processes are characterized by inverse ($\frac{1}{x}$) lines on a $P - V$ graph.

Equation 9.8: Isothermal Process

$$\Delta T = 0$$
$$\Delta PV = 0$$
$$\Delta U = 0$$
$$WD = -\Delta Q$$

Definition 9.13: Isovolumetric / Isochoric Process

An Isovolumetric / Isochoric Process is one where the volume of a system remains constant. These processes are characterized by vertical lines on a $P - V$ graph.

Equation 9.9: Isovolumetric / Isochoric Process

$$\Delta V = 0$$
$$WD = 0$$
$$\Delta U = \Delta Q$$

Definition 9.14: Adiabatic Process

An Adiabatic Process is one where no heat is exchanged with the external system. These processes are characterized by lines which traverse isotherms on a $P - V$ graph.

Equation 9.10: Adiabatic Process

$$\Delta Q = 0$$
$$\Delta U = WD$$

10 Oscillations

10.1 Simple Harmonic Motion

Definition 10.1: Simple Harmonic Motion

Simple Harmonic Motion (SHM) occurs when a body oscillates about a point where its acceleration is proportional to its displacement from said point and directed towards said point.

10.2 SHM Equations

Equation 10.1: SHM Equation

For

$$a = -kx$$
$$x = x_o \cos \omega t$$
$$v = \frac{dx}{dt} = -x_o \omega \sin \omega t$$
$$a = \frac{dv}{dt} = -x_o \omega^2 \cos \omega t$$
$$= -\omega^2 x$$
$$k = \omega^2$$

10.2.1 Graphs of SHM

Bodies undergoing SHM have sinusoidal graphs of displacement, velocity and acceleration due to the trigonometric functions in their equations. Graphs of energy of SHM hold the property of conservation of energy, where the sum of potential energy and kinetic energy, usually in the form of square trigonometric function graphs, always add up to a constant.

10.3 Cases of SHM

10.3.1 Horizontal Oscillations

Horizontal Spring Systems involve a mass connected to a spring. The spring creates a horizontal restoring force.

Pendulum Systems involves a mass connected to a inextensible string. The net force of gravity and tension in the string creates a horizontal restoring force which is assumed to be proportional to horizontal displacement at small angles $\theta < 6°$.

10.3.2 Vertical Oscillations

Vertical Spring Systems involve a mass connected to a vertical spring. The net force of the spring and gravity provides the restoring force which acts on the oscillating mass. The center of oscillations is assumed to be the same position as when the mass is in stationary equilibrium and any SHM acts around this position. Energy is also conserved, where the summation of spring potential and gravitational potential stores energy to be converted into kinetic energy.

10.4 Damping

Definition 10.2: Damping

Damping occurs when there is an external force acting on the object undergoing SHM, usually proportional to velocity (such as air resistance).

Definition 10.3: Light Damping

Light Damping occurs when a damping force causes the amplitude of SHM to decrease exponentially over time.

Definition 10.4: Critical Damping

Critical Damping occurs when a damping force causes a body undergoing SHM to return to equilibrium position and stop oscillating within the shortest possible time.

Definition 10.5: Heavy Damping

Heavy Damping occurs when a damping force causes a body undergoing SHM to return to equilibrium position and stop oscillating over a period of time longer than if the system were critically damped.

10.4.1 Forced Oscillations

Definition 10.6: Forced Oscillations

Forced Oscillations arise when a system has to receive external force in order to undergo SHM.

10.5 Resonance

Definition 10.7: Resonance

Resonance occurs when a system responds to a driving force with a maximum amplitude. This implies the maximal transfer of energy between driving and driven systems, therefore implying that the driving frequency is equal to the natural frequency of the driven system.

Definition 10.8: Natural Frequency

The Natural Frequency, also known as the Resonant Frequency or Resonance Frequency, is the frequency at which a system is able to receive energy at a maximum.

RI's tutorial solutions have used the terms "Resonance Frequency", "System at Resonance" and "Resonance Occurs". Use these if phrasing is vague and if you worry about presentation / phrasing marks.

11 Wave Motion

Definition 11.1: Wave

Waves are displacements in a system where oscillations in a medium are propagated and therefore energy is propagated without physically relocating the medium.

11.1 Transverse and Longitudinal Waves

Definition 11.2: Transverse Wave

Transverse Waves involve particles oscillating in a direction perpendicular to the direction of energy transfer.

Definition 11.3: Longitudinal Wave

Longitudinal Waves involve particles oscillating in a direction parallel to the direction of motion of energy transfer.

11.2 Intensity of Waves

Definition 11.4: Intensity

Intensity is the quantization of how much energy is transferred to a surface area by a wave per unit time, measured in $\mathrm{W\,m^{-2}}$

Equation 11.1: Intensity

For power P spread across surface area S, or a proportionality constant k, frequency f and amplitude of wave A, intensity I is defined as

$$I = \frac{P}{S} = kf^2A^2$$

As wave energy dissipates across a larger surface area and its total power remains the same, its intensity decreases. Cylindrically radiating waves have $S = 2\pi r$ and follow a inverse rule while spherically radiating waves have $S = 4\pi r^2$ and follow a inverse square rule.

11.3 Polarization

Equation 11.2: Polarization

For a angle θ between the original and new planes of polarized light, the amplitude of wave A and intensity of wave I are changed as:

$$A\prime = A\cos\theta$$
$$I\prime = I\cos^2\theta$$

12 Superposition

12.1 Principle of Superposition

Definition 12.1: Principle of Superposition

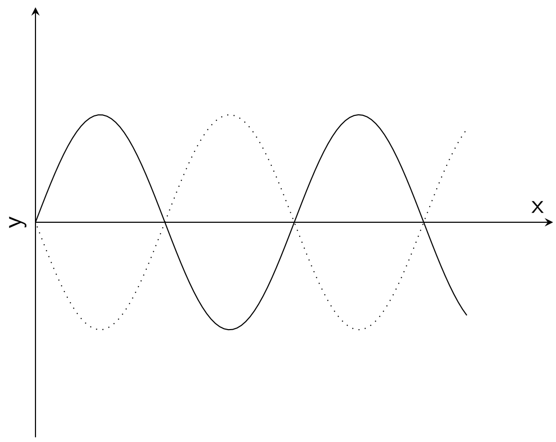
The Principle of Superposition states that the effect of two stimuli at a point is the sum of the two responses.

12.2 Stationary Waves

Definition 12.2: Stationary Waves

Stationary Waves result from the superposition of two progressive waves of same frequency, amplitude and speed traveling along the same line but in opposite directions.

The resultant stationary wave has zero net energy transfer, and is drawn as the envelope of the displaced particles, with one waveform drawn with a solid line and one waveform with a dotted line.



Definition 12.3: Node

The Node is a location in a stationary wave where the particles no longer oscillate, typically when two waves reach the point π radians out of phase.

Definition 12.4: Antinode

The Antinode is a location in a stationary wave where the particles oscillate with the maximum displacement, typically when two waves reach the point in phase.

Depending on the environment in which a stationary wave is produced determines the distribution of nodes and antinodes. Since the distribution of nodes and antinodes determine the wavelength of a stationary wave and the speed of the wave in the propagation medium is typically constant, the frequencies at which a stationary wave is produced is now determined.

Definition 12.5: Hard Boundary

A Hard Boundary is able to supply a restoring force which then causes a reflected wave to undergo a π radian phase shift (due to conservation of energy). A Hard Boundary in a stationary wave creates a node.

Definition 12.6: Soft Boundary

A Soft Boundary is unable to supply a restoring force and hence causes a wave to reflect with the same polarity back. A Soft Boundar in a stationary wave creates an antinode.

Definition 12.7: Fundamental Frequency

The Fundamental Frequency of a stationary wave is the lowest frequency at which a stationary wave can be produced.

Definition 12.8: Harmonic

The n^{th} Harmonic of a wave is the frequency where a stationary wave can be produced where $n = \frac{f_{\text{hamonic}}}{f_{\text{fundamental}}}$

Definition 12.9: Overtone

The n^{th} Overtone of a wave is the nth next highest frequency than the fundamental frequency that a stationary wave can be produced.

To produce a stationary wave, the relationships between need to be maintained.

Equation 12.1: Frequency for two-node or two-antinode Systems

Where v is the speed of wave in a system, L is the length of system and n is the overtone number, the fundamental frequency for stationary wave f is

$$f = \frac{(n+1)v}{2L}$$

Equation 12.2: Frequency for one-node one-antinode Systems

Where v is the speed of wave in a system, L is the length of system and n is the overtone number, the fundamental frequency for stationary wave f is

$$f = \frac{(2n+1)v}{2L}$$

12.2.1 Stationary Waves in String

Ends of string fixed to a stationary point behave like hard boundaries and create nodes. Ends of string which can oscillate perpendicularly to the direction of the wave (by means of a guide rail or some similar mechanism) behave like soft surfaces and create antinodes.

The speed of the wave in string v depends on the material of the string and is proportional to the tension on the string.

12.2.2 Stationary Waves in Pipes

Definition 12.10: End Correction

The End Correction e of a stationary wave in a pipe is an extra effective length added to open ends, which arises because a antinode occurs slightly outside the end of a pipe.

Closed ends of pipes behave like hard boundaries and create nodes. Open ends of pipes behave like soft surfaces because pressure inside a pipe is partially reflected when it comes into contact with an external system, creating antinodes which are one end correction away from the edge of the pipe. Pipes with two open ends have to account for two end corrections.
The speed of the wave in a pipe v depends on the speed of wave in the medium inside the pipe, such as the speed of sound in air $334\,\mathrm{m\,s^{-1}}$ for pipes in air.

12.2.3 Diffraction

Definition 12.11: Diffraction

Diffraction is where waves bend when passed through an aperture of comparable length to its wavelength or when passing around an obstacle.

For the case of an aperture (also known as Fraunhofer diffraction), waves radiate in a radial fashion.

Definition 12.12: Huygen's Diffraction

Diffraction occurs as points on a wavefront can be treated as secondary sources of wavelets, where the envelope of the secondary wavelets form the next wavefront of the original wave.

12.3 Interference

Definition 12.13: Coherence

Two waves or sources are coherent if they have a constant phase difference, implying equal frequency, speed and wavelength.

Definition 12.14: Interference

Interference between two waves occurs when both reach a point in space, where their resultant effect is obtained through the principle of superposition, hence the effective displacement is the vector sum of the displacement due to each waves.

Definition 12.15: Constructive Interference

Destructive Interference occurs when two coherent waves reach a point in phase to form a resultant maximum displacement.

Definition 12.16: Destructive Interference

Destructive Interference occurs when two coherent waves reach a point with a phase difference of π radians to form a resultant minimum displacement.

Definition 12.17: Fringe

A Fringe is a location at which constructive (bright fringe) or destructive (dark fringe) interference can be observed.

12.3.1 Double Source Interference

Definition 12.18: Path Difference

The Path Difference is the difference in the distance that each wave travels fro its source to the point where two waves meet.

Definition 12.19: Order

The Order of a fringe is the rounded-up absolute path difference divided by wavelength between two waves which cause the formation of a fringe.

Two coherent sources of radial waves produce a interference pattern of dark and bright fringes on a plane far away from the two sources.

In the case of a double-slit experiment where the distance between source and observation screen is sufficiently large and the angle between 0^{th} order maximum and other fringes are small enough to use small angle approximation, the distance between two successive bright or dark fringes is assumed to be constant.

Equation 12.3: Two-Slit Experiment Fringe Septation

For some wavelength λ, distance between source and screen D and separation of sources a, the separation between two fringes x is

$$x = \frac{\lambda D}{a}$$

12.3.2 Diffraction Grating

Definition 12.20: Diffraction Grating

A Diffraction Grating is a sheet of material with multiple apertures on its surface. In optics, typical diffraction gratings have around 100 to 1000 apertures per mm.

Light passed through a Diffraction Grating creates sharp bright fringes.

Equation 12.4: Condition for Constructive Interference

For some slit separation d, wavelength λ, non-zero integer n and angular displacement θ_n, bright fringes form where the following equation is satisfied

$$d\sin\theta_n = n\lambda, \quad n < \frac{d}{\lambda}$$

12.3.3 Single Source Interference

A single source of waves when observed on a screen forms a unique interference pattern. The center of the pattern is a diffuse section due to diffraction. However, when considering the aperature source as multiple coherent sources of waves, there arises a difference in path difference which then causes minima to form.

Equation 12.5: Condition for Single-Source Minima

For some opening of length b, wavelength λ and angular displacement from source θ, single-source minima form where

$$\sin\theta = \frac{\lambda}{b}$$

12.4 Rayleigh Condition

Definition 12.21: Resolution of Images

Two images are clearly resolved when they are distinguishable from each other.

Equation 12.6: Rayleigh Condition

For a image to just be resolved when observed through an aperture, angular separation (crossing both sides of the medial line) θ, wavelength λ and size of aperture b

$$\sin\theta = \frac{\lambda}{b}$$

$$\theta \approx \frac{\lambda}{b}$$

For a circular aperture, $\theta \approx 1.22\frac{\lambda}{b}$.

The Rayleigh Condition was arbitrarily set such that the peak intensity from one image intersects with the minimum of another image. Should the images be any closer the combination of the two graphs will form one single peak (and hence the images are indistinguishable) and should they be any further away the peaks will be more resolved.

13 Electric Field

Equation 13.1: Electric Force Equations

For a charge q_1, the radius from it r, and the permittivity of free space ϵ_0, the force experienced F by a secondary charge q_2, the strength of electric field g, the potential energy U of a secondary charge q_2 and the potential V are given by the equations

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$E = \frac{q_1}{4\pi\epsilon_0 r^2}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$V = \frac{q_1}{4\pi\epsilon_0 r}$$

13.1 Electric Charge

Definition 13.1: Coulomb C

The Coulomb C is the SI unit for electric charge. One coulomb is the amount of charge passing through one point with current 1 A.

Definition 13.2: Electric Charge

13.2 Electric Force

Definition 13.3: Coulomb's Law

Coulomb's Law states that the force between two charges is proportional to the product of both charges and inversely proportional to the square of the distance between them.

An electric charge in the proximity of another electric charge experiences electric force.

13.3 Electric Field

Definition 13.4: Electric Field

The Electric Field is a space where a charge experiences an electric force, with units $\mathrm{V\,m^{-1}}$.

Definition 13.5: Electric Field Strength E

The Electric Field Strength E at a point in space is the force per unit positive charge at a point.

Electric field lines in a diagram indicate the direction of acceleration of a positive test charge at a specific point. Electric field lines will never cross each other. A higher density of field lines in a unit area indicates a stronger electric field.

Electric field lines originate from positive charges and terminate at negative charges. Field lines will also always be perpendicular to any conducting metal body since any other electric field will induce movement in the charges in the metal.

For a hollow sphere with charge distributed at its surface, the net electric field inside the sphere is 0.

13.4 Electric Potential

Definition 13.6: Electric Potential V

The Electric Potential V at a point is the work done per unit positive charge by an external force in bringing a small test charge from infinity to a point, with units V.

Definition 13.7: Potential Difference ΔV

The Potential Difference ΔV is the change in electric potential between two points.

Equipotential lines are lines in a diagram where all points have the same potential. Equipotential lines are always perpendicular to electric field lines.

13.5 Parallel Charged Plates

For two long parallel charged plates separated by distance $d$with potential difference V between them, the electric field between these two plates are approximately constant across the plates. As a result, the electric field between them is given by the equation $E = \frac{V}{d}$.

14 Current of Electricity

14.1 Current

Definition 14.1: Current I

Current I is the rate of flow of charge, measured in A.

Definition 14.2: Coulomb C)

The Coulomb C is the number of electrons passing through a point in 1 s with a current of 1 A.

14.1.1 Derivation of Current

Equation 14.1: Current by Considering Charge Carriers

For a conducting medium with n charge carriers per unit volume, each of charge q, their drift velocity v and the medium's cross sectional area A, the current I is given by the equation

$$\begin{aligned} I &= \frac{\text{Charge passing}}{\Delta t} \\ &= \frac{\text{Volume} \times nq}{\Delta t} \\ &= \frac{(Av\Delta t)nq}{\Delta t} \\ &= nAvq \end{aligned}$$

14.2 Potential Difference and EMF

Definition 14.3: Potential Difference V

The Potential Difference between two points in a circuit is the electrical energy converted to other forms of energy per unit charge as they pass between these points, measured in V.

Equation 14.2: Potential Difference V

For a circuit with work done W and current I, the potential difference V is given by the equation

$$V = \frac{W}{I}$$

Definition 14.4: Volt

One Volt is the potential difference between two points where a charge of 1 C will convert 1 J of electrical energy to other forms.

Definition 14.5: Electromotive Force ϵ

The Electromotive Force ϵ is the amount of other forms of energy converted into electrical energy per unit charge when driving a charge throughout the circuit.

14.3 Resistance

Definition 14.6: Ohm's Law

Ohm's Law states that the potential difference across a circuit component is proportional to the current passing through it, with proportionality constant R.

Equation 14.3: Ohm's Law

For a circuit component with resistance R and current passing through I, the potential difference V over the component is given by the equation

$$V = RI$$

Definition 14.7: Resistance R

Resistance R is the ratio of potential difference to current passing through a circuit component.

Definition 14.8: Ohm Ω

One ohm is the resistance of a conductor when 1 V of potential difference induces a current of 1 A across it.

Definition 14.9: Resistivity ρ

The Resistivity ρ of a material is the constant of proportionality between the resistance of a component and the ratio of its cross-sectional area to its length.

Equation 14.4: Resistivity ρ

For a material of resistivity ρ, length l and area A, its resistance is given by the equation

$$R = \frac{\rho l}{A}$$

14.4 Power

Definition 14.10: Power P

The Power dissipated by a circuit component is the rate of which energy is dispersed by it.

Equation 14.5: Power P

For a component with resistance R, current passing through I and potential difference across it V, the power dissipated by it is given by the equation

$$P = IV = I^2R = \frac{V^2}{I}$$

Definition 14.11: Internal Resistance

A non-ideal power source has an Internal Resistance in series with its induced electromotive force.

Definition 14.12: Efficiency η

The Efficiency of an electric system is the ratio of useful power dissipated to total power dissipated in a circuit in percent.

Equation 14.6: Efficiency η

$$\begin{aligned}\eta &= 100\% \times \frac{P_{\text{useful}}}{P_{\text{total}}} \\ &= 100\% \times \frac{V_{\text{load}}}{V_{\text{total}}} \\ &= 100\% \times \frac{R}{R+r}\end{aligned}$$

Definition 14.13: Maximum Power Theorem

For a power source with internal resistance r and external load R, the largest power is dissipated through the external load when $r = R$.

14.5 Types of Conductors

Definition 14.14: Ohmic Conductor

An Ohmic conductor is a conductor which has a constant resistance regardless of other physical conditions.

Definition 14.15: Metal Conductor / Filament Bulb

A Metal Conductor or Filament Bulb is a conductor whose resistance increases as temperature increases.

At higher temperatures, the cations in a metal vibrate faster and are hence more likely to collide with current-carrying electrons. This results in a net effect where more collisions occur and the electrons passing through face more resistance, hence the resistance increases.

Definition 14.16: Semiconductor

A Semiconductor is a conductor whose resistance decreases as temperature increases.

At higher temperatures, more charge carriers in a semiconductor are able to disassociate and carry a current, hence its resistance decreases.

15 D.C. Circuits

15.1 Combining Resistance

Equation 15.1: Resistance in Series

For multiple resistors $R_1, R_2 \ldots$ connected along each other, their total resistance R is given by the equation

$$R = R_1 + R_2 + \ldots$$

Equation 15.2: Resistance in Parallel

For multiple resistors $R_1, R_2 \ldots$ connected beside each other, their total resistance R is given by the equation

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$$

15.2 Potential Divider Circuit

Definition 15.1: Potential Divider

A Potential Divider circuit is an arrangement of two or more resistors connected in series across a voltage supply in order to obtain a usable potential difference that is a fraction of the voltage supply.

The output voltage of a potential divider circuit is dependent on the resistance between the terminals of the output.

Equation 15.3: Potential Difference from Potential Divider

If a voltage supply of electromotive force V is connected to two resistors R_1 and R_2, a output terminal across R_1 will have output voltage V_O given by the equation

$$V_O = V\left(\frac{R_1}{R_1 + R_2}\right)$$

To obtain various voltages, a variable resistor such as a rheostat with its sliding contact can be used instead of fixed resistors.

15.3 Potentiometer

Definition 15.2: Potentiometer

A Potentiometer is a circuit used to measure the magnitude of an unknown electromotive force by passing a current through a known potential divider circuit.

Definition 15.3: Galvanometer

A Galvanometer is a sensor used to determine small flows in current and is used to identify balance lengths where there is no current passing through it, i.e. used to find two points whose potential difference is 0.

Definition 15.4: Balance Length

The Balance Length is the position on a resistance wire where the current passing through a galvanometer attached to that point and another point on an external circuit is zero.

Given the balance length of a potentiometer setup, two equipotential points are identified and can be used to calculate resistances and potential differences in either circuit. If two parts of the potentiometer circuit have no current passing between them, they can be considered as disconnected circuits.

15.4 Kirchoff's Laws (Out of syllabus)

Definition 15.5: Kirchoff's First Law

Kirchoff's First Law states that at any point in a circuit there is zero net current flow in and out of that point.

Definition 15.6: Kirchoff's Second Law

Kirchoff's Second Law states that if a closed loop is identified in a circuit, the sum of emf in the loop is equal to the sum of the IR products across each component in the loop.

Given the first law, arbitrary current variables $I_1, I_2 \ldots$ can be assigned to sections of the circuit and can be associated through a series of sums. Once sufficient loops are identified and their IR products accounted for, the currents, voltages and resistances in a circuit can be easily algebraically solved.

16 Electromagnetism

16.1 Magnetic Fields

Definition 16.1: Magnetic Field

A Magnetic Field is a region in which a magnetic pole experiences a force.

Magnetic fields are induced by permanent magnets as a result of atomic dipoles and electric currents on a quantum scale whereas a current of electricity can also induce magnetic fields from an electromagnet. The direction of magnetic field is determined by the orientation of dipoles in a permanent magnet and the direction of current in a electromagnet.
Magnetic fields lines in a diagram are drawn from North to South and indicate the direction of force on a test north pole. As with electric fields, like field lines repel and unlike field lines attract

Definition 16.2: Magnetic Flux Density

The Magnetic Flux Density of a magnetic field quantities force per unit length of a long straight conductor carrying current at right angles to the magnetic field, measured in Tesla T or $\mathrm{N\,A^{-1}\,m^{-1}}$.

Equation 16.1: Biot-Savart Law (not in syllabus)

The general law for calculation of magnetic field density B for a wire with path L and current I at a point r is given by the equation

$$B = \frac{\mu_0}{4\pi} \int \frac{I d\overrightarrow{L} \times \hat{r}}{r^2}$$

The Magnetic Flux Density of a region describes the magnitude of a magnetic field.

Definition 16.3: Tesla

One Tesla of magnetic flux in a region acting perpendicular to a long straight wire carrying a current of 1 A will experience a force per unit length of $1\,\mathrm{N\,m^{-1}}$.

16.1.1 Magnetic Field due to Long Wire

Definition 16.4: Right Hand Rule

In relation to a curled up right hand with its thumb pointing away from the fingers, a current moving in the direction of the thumb induces a magnetic field in the circular direction of the fingers. Alternatively, a solenoid with its current moving in the circular direction inscribed by a curled hand's fingers will induce magnetic field lines indicated by the direction the thumb is pointing in.

Equation 16.2: Magnetic Field due to Long Wire

For a long wire distance r away from a point carrying current I, its magnetic field strength B at said point is given by the equation

$$B = \frac{\mu_0 I}{2\pi r}$$

16.1.2 Magnetic Field due to Ring

Equation 16.3: Magnetic Field due to Ring

For a conducting ring of radius r and number of turns of wire N carrying current I, its magnetic field strength B at the center of the coil is given by the equation

$$B = \frac{\mu_0 N I}{2r}$$

16.1.3 Magnetic Field due to Solenoid

Equation 16.4: Magnetic Field due to Solenoid

For a solenoid of turns per meter n carrying a current I, its magnetic field strength B inside the coil is given by the equation

$$B = \mu_0 n I$$

16.2 Magnetic Force

Definition 16.5: Fleming's Left Hand Rule

For a left hand with its thumb and two forefingers pointing perpendicular to each other, orienting two out of the following three directions will give the direction of the last component: thumb pointing in direction of Force, forefinger in the direction of magnetic field and middle finger in the direction of current.

Unlike magnetic field inducing currents which typically require some arrangement of an infinitely long wire or a current in a loop, magnetic force in a current carrying wire can be calculated for some finite length of wire.

Magnetic force is only dependent on current which is perpendicular to the direction of the magnetic field, hence a term of $\sin\theta$ is present in most calculations to account for currents which may vary in angle to the external magnetic field.

16.2.1 Magnetic Force on Current-carrying Conductor

Equation 16.5: Magnetic Force on Current-carrying Conductor

For a wire with length L carrying current I in a magnetic field B and with angle between current and magnetic field θ, the force experienced F is given by the equation

$$F = BIL\sin\theta$$

Two wires with currents flowing in the same direction experience attractive forces whereas wires with currents flowing in the opposite direction experience repulsive forces. From observing these forces, the quantitative value of the SI ampere can be fixed.

Definition 16.6: Ampere (not in syllabus)

One Ampere of current is the current carried in two infinite current-carrying wires of negligible cross-section such that there is a force of 2×10^{-7} N m^{-1} between them.

16.2.2 Magnetic Force on Free Charge

Equation 16.6: Magnetic Force on Free Charge

For a free charge q with velocity v in a magnetic field B and with angle between velocity and magnetic field θ, the force experienced F is given by the equation

$$\begin{aligned} F &= BIL\sin\theta \\ &= B\frac{q}{t}L\sin\theta \\ &= Bq\frac{L}{t}\sin\theta \\ &= Bqv\sin\theta \end{aligned}$$

A free charge moving in the presence of a magnetic field may experience force due to the magnetic field and the effective 'current' which it creates. For a charge with some component of velocity perpendicular to magnetic field $v_\perp$, it will experience some force perpendicular to its direction of motion and hence experience circular motion perpendicular to the magnetic field. If its velocity has a component parallel to magnetic field $v_\parallel$ as well, it will exhibit helical motion / move in a helix.

Definition 16.7: Specific Charge

The specific charge of a charged particle is the ratio of its charge to its mass.

Equation 16.7: Lorentz Force

The total force F experienced by a charge q in an electric field E and magnetic field B while moving in velocity v is given by the equation

$$F = q(\vec{E} + \vec{v} \times \vec{B})$$

17 Electromagnetic Induction

17.1 Flux and Flux Linkage

Definition 17.1: Magnetic Flux

The Magnetic Flux of a surface is the product of the magnetic field strength and the area of the surface perpendicular to the magnetic field, with units Wb or T m^{-2}

Equation 17.1: Magnetic Flux

The Magnetic Flux Φ of a surface with area A, magnetic field strength B and angle between normal of surface and magnetic field θ is given by the equation

$$\Phi = AB\cos\theta$$

Definition 17.2: Magnetic Flux Linkage

The Magnetic Flux Linkage of a coil is the product of the flux through the coil and the number of turns on the coil.

Equation 17.2: Magnetic Flux Linkage

The Magnetic Flux Linkage λ of a coil with N turns and magnetic flux through cross section Φ is given by the equation

$$\lambda = N\Phi$$

Definition 17.3: Weber

One Weber is the amount of magnetic flux through an area of $1\,\text{m}^2$ with magnetic field strength perpendicular to the area of $1\,\text{T}$

17.2 Induced Electromotive Force

Definition 17.4: Faraday's Law of Electromagnetic Induction

Faraday's Law of Electromagnetic Induction states that an induced electromotive force in a system is proportional to this rate of change of magnetic flux linkage is formed.

Equation 17.3: Faraday's Law of Electromagnetic Induction

The induced electromotive force E of a system with changing flux linkage Φ over time t is given by the equation

$$E = \frac{dE}{dt}$$

Definition 17.5: Lenz's Law

Lenz's Law states that the direction of an induced current will act to oppose the change in magnetic flux linkage.

17.3 Moving Metals and EMF

17.3.1 Moving Charged Rod

A charged metallic rod of length l is placed in a region with magnetic field B perpendicular to its length and moved at speed v in a direction perpendicular to its length and the magnetic field. As an area of lvt in a unit of time, magnetic flux linkage thus changes and a potential difference between the two terminals of the rod $E = Blv$ is induced.

17.3.2 Rotating Metal Disc

A metallic disc with area A is placd horizontally in a magnetic field B normal to it is rotated at a frequency f. As an amount of magnetic flux $\Phi = AB$ is swept each rotation, there is a potential difference between the center of the disc and its edge equal to $E = ABf$.

18 Alternating Current

18.1 Alternating Current

Definition 18.1: Alternating Current

An alternating current is a current which periodically reverses direction of current flow when transmitting power.

Equation 18.1: Sinusoidal Power Source

A Sinusoidal Power Source with peak current I_0, peak voltage V_0 and frequency f has instantaneous current I and voltage V at time t given by the equations

$$I = I_0 \sin(2\pi f t)$$
$$V = V_0 \sin(2\pi f t)$$

Keep in mind that power varies at twice the frequency of the current or voltage in a sinusoidal source.

18.1.1 Root Mean Square

Definition 18.2: Root Mean Square Value

The Root Mean Square value of a quantity of a system with alternating current is the equivalent direct current value where thermal energy is released at the same rate when passed through a resistor.

Equation 18.2: Mathematical Notation for RMS

For some variable X which may or may not change with time

$$X_{\text{rms}} = \sqrt{\langle X^2 \rangle}$$

The root mean square (rms) of a function is a single value describing the square root of the mean of the square of the function. The rms value of a system with alternating current is useful as it is a more useful quantification of the 'average' voltage or current than its numerical mean.

Equation 18.3: RMS of Sinusoidal Power Source

The root-mean-square current I_{rms} , root-mean-square voltage V_{rms} and root-mean-square power $P_{\text{rms}}$of a Sinusoidal Power Source with peak current I_0 and peak voltage V_0 is given by the equation

$$I_{\text{rms}} = \frac{I}{\sqrt{2}}$$
$$V_{\text{rms}} = \frac{V}{\sqrt{2}}$$
$$P_{\text{rms}} = \frac{P}{2}$$

Note that any rms value is independent of the frequency of the system and valid at any point in time.

18.2 Transformers

A Transformer is a device which uses electromagnetic induction to step up or step down the voltage of a power source to a power sink.

Transformers typically comprise two wire coils around a laminated soft iron core. When alternating current is passed through the primary core, magnetic field changes around the iron core. The secondary core experiences the changing magnetic field and produces an electromotive force depending on the ratio of the number of turns of wire in either coil.

Equation 18.4: Ideal Transformer Values

An ideal transformer with turns N_P and N_S have their current I and potential difference V related by the equations

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$
$$V_P I_P = V_S I_S$$

Typical transformers are not ideal:

- Primary and secondary coils have non-zero resistance and may lose energy to heat
- Eddy currents may form in the iron core which lead to heat loss. Iron cores are hence laminated by layering up multiple sheets of iron to prevent the formation of such currents.
- Magnetic field lines will not be completely linked across the transformer and will not result in 100% power transmission.

18.3 Power Transmission

Alternating current is more often used to transmit power over long distances:

- Alternating current is more easily stepped up or down to various voltages for different uses
- Power transmission at high voltages leads to lesser power losses. Alternating current is hence advantageous in that it is easy to step it up to be transmitted.

18.4 Rectification

Definition 18.3: Rectification

Rectification is the process of turning an alternating current power source to a direct current power source.

A alternating current can be half-wave rectified or full-wave rectified using diodes which limit the direction at which current can flow.

Equation 18.5: RMS of Half Wave Rectified Sinusoidal Power Source

$$I_{\text{rms}} = \frac{I}{2}$$
$$V_{\text{rms}} = \frac{V}{2}$$
$$P_{\text{rms}} = \frac{P}{4}$$

19 Quantum Physics

19.1 Photoelectric Effect

Definition 19.1: Photoelectric Effect

The Photoelectric Effect is the phenomena where free electrons are emitted from a metal surface when electromagnetic radiation of sufficiently high energy is incident on the surface.

Four key observations were made from this experiment:

1. There exists a minimum threshold frequency f_0 for electrons to be emitted
2. No time delay was observed between when the metal was exposed to light and when a current was detected
3. Stopping potential V_S and therefore maximum kinetic energy of electrons was independent of intensity but dependent on frequency
4. Current of electrons was proportional to intensity

19.1.1 Failures of Classical Theory

Classical wave theory considers light as a wave, which involves continuous energy transfer which is dependent on the intensity of light.

According to classical wave theory, several contradictory observations from the photoelectric effect experiment are made:

- A light of high intensity but frequency below threshold will theoretically be able to eject electrons due to higher energy transfer, but instead no electrons are ejected.
- A light of high intensity and with sufficient frequency will not increase the stopping potential, despite electrons theoretically having received more energy.
- A light of low intensity and with sufficient frequency will theoretically require some time to absorb sufficient energy to be ejected, but instead electrons are always ejected instantaneously.

19.1.2 Quantum Theory of Light

Definition 19.2: Photon

A Photon is a quantum of electromagnetic energy.

Equation 19.1: Energy of Photon

The Energy E of one Photon with frequency f, where h is Planck's constant, is given by the equation:

$$E = hf = h\frac{c}{\lambda}$$

By suggesting that photons of a specific frequency carries and transfers energy in discrete packets of energy hf, the photoelectric effect can be modeled as the result of photons colliding and interacting with electrons in the metal.

As a photon is incident on an electron, it transfers all of its energy to that electron as kinetic energy. Electrons then may collide with other particles in the metal and dissipate energy as heat, or can escape the attractive forces of the metal lattice and be ejected from the metal with remaining kinetic energy, which is then observed as photocurrent.

Definition 19.3: Work Function Energy Φ

The Work Function Energy Φ of a metal is the minimum amount of energy required for a free electron to escape from the surface of the metal.

Equation 19.2: Einstein's Photoelectric Equation

Upon a collision of a photon with energy hf, the kinetic energy $\frac{1}{2}m_e v^2$ of a resulting photoelectron exiting from a metal of work function $\Phi$is determined by the equation

$$hf = \Phi + \frac{1}{2}m_e v^2$$

Quantum theory predicts the existence of threshold frequency as this indicates a minimum energy for the formation of a photocurrent, caused by the presence of the work function Φ. Any photon with energy less than Φ is unable to form a free electron with non-zero kinetic energy, where its collided electron will be unable to escape the metal.

According to quantum theory, varying the intensity of a light only changes the number of photons transmitted and does not actually change the amount of energy transferred from one photon to one electron in a single collision. As a result, increasing the intensity of a light source of insufficient frequency will not affect the maximum energy which an electron receives and thus still will not create a photocurrent, and increasing the intensity of a light source of sufficient frequency will only increase the number of electrons in the photocurrent rather than the maximum kinetic energy of the electrons.

19.2 Wave-particle Duality

Equation 19.3: de Broglie Wavelength

Any wave or particle with either a wavelength λ or momentum p respectively has an effective momentum or wavelength given by the equation:

$$\lambda = \frac{h}{p}$$

An object with a large de Broglie wavelength behaves like a wave whereas an object with a small de Broglie wavelength behaves like a wave. As a result, electrons with sufficient kinetic energy have a large enough de Broglie wavelength to follow rules of diffraction and interference, and can be observed under a setup for Electron Diffraction.

19.3 Absorption and Emission Spectra

When white light is incident on a gas of an element, specific frequencies of light are wholly absorbed by the gas and then radiated in all directions. When a gas is heated, it will only radiate light of specific frequencies. These specific wavelengths which an atom is more able to absorb and emit form the adsorption and emission spectra of a gas.

These special wavelengths of light arise as each atom has specific 'energy transitions' of which electrons can move from one stable energy state to another after absorbing or emitting energy (in the form of a photon).

Each atom has several allowed orbits, each with a corresponding number n and a corresponding energy E_i of the electron in that orbit. When a photon of sufficient energy collides with an electron, the electron may absorb all of the photons' energy and be excited to a higher energy orbit. Later, if there is a empty orbital in a lower energy orbit, an excited electron may demote itself to the lower energy level while emitting a photon to release said energy.

Definition 19.4: Ground State

The Ground State of an electron in an atom is the lowest energy level $(n = 1)$possibly held by a stable electron in that atom.

Equation 19.4: Transition Energy

A transition from the initial energy level E_i to final energy level E_f involves the emission of a photon with energy hf OR the absorption of a photon with at least energy hf.

For Emission:

$$E_i - E_f = hf$$

For Adsorption:

$$E_f - E_i \leq hf$$

Definition 19.5: Ionization Energy

The Ionization Energy of an atom is the minimum energy required to remove an electron completely from the atom, from ground state $n = 1$ to the infinite level $n = \infty$.

19.3.1 X-ray Spectra

When high-speed electrons are incident on a metal sheet, high energy X-rays are then emitted.

The graph of relative intensity against wavelength has two key components. The continuous spectrum is a smooth curve that rises from zero, reaches a peak and then approaches zero as it extends to infinity. Two characteristic peaks are also observed, which arise when electrons dislodge electrons in the $n = 1$ orbital of a metal atom, allowing for $n = 2 \rightarrow n = 1$ and $n = 3 \rightarrow n = 1$ transitions to occur which then emit X-rays at much higher intensities.

A minimum wavelength exists as this represents the highest possible energy of an emitted photon, which is given off when an approaching electron converts all its energy to a photon in a single collision. Lower energy photons are created in less effective collisions which give rise to the continuous spectrum, but no photon can have a higher energy than the maximum kinetic energy of the approaching electrons.

Since the characteristic peaks are an innate property of the metal, varying the energy and number of incident electrons will not affect the wavelengths of the characteristic peaks. The exact wavelengths of the characteristic peaks can also be used to identify what metal the electrons are incident upon.

19.3.2 Heisenberg Uncertainty Principle

Definition 19.6: Heisenberg Uncertainty Principle

The Heisenberg Uncertainty Principle states that for any particle, the error in the simultaneous measurement of its location Δx and its momentum in said direction Δp_x (and therefore wavelength) must be larger or approximately equal to h.

Equation 19.5: Heisenberg Uncertainty Principle

In a simultaneous measurement of a particle's location and momentum, the uncertainties in the location Δx and momentum p_x related by the limit:

$$\Delta x \Delta p_x \geq h$$

20 Nuclear Physics

20.1 The Nucleus

Definition 20.1: Nucleus

The Nucleus is a dense region at the center of an atom which contains its protons and neutrons.

Definition 20.2: Nucleon

A Nucleon refers to particles found in the nucleus, which can be either protons or neutrons.

Definition 20.3: Mass Number A

The Mass Number A, also known as Nucleon Number, of a nucleus is the total number of protons and neutrons in a nucleus.

Definition 20.4: Proton Number Z

The Proton Number Z of a nucleus is the total number of protons in a nucleus.

Definition 20.5: Nuclide

A Nuclide is a specific type of nucleus with a specific mass number and a specific proton number.

Definition 20.6: Isotope

Isotopes are a group of nuclides with same proton number but different mass number.

20.1.1 Nuclear Mass

Definition 20.7: Atomic Mass Unit u

1 Atomic Mass Unit , written 1 u, is equal to $\frac{1}{12}$ the mass of a $^{12}_{6}C$ atom.

A proton has a mass of 1.007 277 u, a neutron has a mass of 1.008 665 u and an electron has a mass of 0.000 549 u.

These values are not a perfect '1' due to binding energy mass defect.

20.1.2 Rutherford Scattering Experiment

In the Rutherford Scattering Experiment, a gold foil was bombarded with helium nuclei and the directions of which helium nuclei were deflected were recorded. A large majority of helium nuclei would seem to pass through the gold foil while a few helium nuclei (1 in 8000) would be deflected to the side.

Deflections of helium nuclei to the side are caused by electric repulsion between the positively charged helium nuclei and the positively charged gold nuclei. The closer the helium nuclei approaches the gold nuclei, the more repulsion is experienced and the exit path of the helium nuclei is deflected more backward.

A large majority of helium nuclei passed through undeflected as they travel through the large amount of empty space between gold nuclei, whereas only a few come near gold nuclei and are deflected. This is indicative of the extremely small size of nuclei relative to their atomic radii.

20.2 Mass-Energy Equivalence

Einstein's postulates on relativity and quantized light lead to the conclusion that mass and energy are interchangeable quantities.

Equation 20.1: Einstein's Mass-Energy Equivalence

An amount of energy E is energetically equivalent to a mass m and vice versa by the equation

$$E = mc^2$$

20.3 Binding Energy and Mass Defect

When nuclei are formed from protons and neutrons, large amounts of energy are released due to the strong nuclear force. This release of energy is accompanied with a reduction in observed mass of the entire atom.

Definition 20.8: Mass Defect

The Mass Defect of a nucleus is the difference between the sums of individual nucleons and the total mass of the final nucleus.

Equation 20.2: Mass Defect

For a nucleus with mass m, number of protons A and number of neutrons N, its mass defect Δm is given by the equation

$$\Delta m = Am_p + Nm_n - m$$

Definition 20.9: Nuclear Binding Energy

The Nuclear Binding Energy of a nucleus is the energy required to separate its nucleons to infinity. This is numerically equivalent to the energy of its mass defect.

20.4 Nuclear Reactions

Definition 20.10: Binding Energy per Nucleon

The Binding Energy per Nucleon is the energy required to separate its nucleons to infinity divided by the number of nucleons in a nucleus.

A larger binding energy per nucleon indicates that a nucleus is more stable. As a result, reactions of multiple nuclei to form a nucleus with larger binding energy per nucleon are energetically feasible processes, and can be used to obtain energy.

The highest binding energy per nucleon resides between Fe and Ni nuclei.

Definition 20.11: Nuclear Reaction

A Nuclear Reaction is a process where two nuclei or other subatomic particles collide to form new species of nuclides.

Definition 20.12: Fission

Definition 20.13: Nuclear Fission

Nuclear Fission is the splitting of a heavy nucleus to two lighter nuclei of approximately the same mass.

Splitting heavy parent nuclei releases a large amount of energy as the daughter nuclei are typically more stable.

Nuclear Fission is typically triggered through bombardment of the heavy nuclei with a high-speed neutron and hence require energy input to begin. However, since many fission reactions also produce high-energy neutrons, each reaction can cause additional reactions, increasing the rate of reaction in an exponential fashion in a 'chain reaction' which will release large amounts of energy.

Such methods of releasing energy are harnessed for energy production (nuclear energy) and military usage (atomic bombs).

Definition 20.14: Fusion

Definition 20.15: Nuclear Fusion

Nuclear Fusion is the combination of two light nuclei to form a nucleus of greater mass.

Combining nuclei of very low mass such has Hydrogen and Helium produce even more energy per nucleon than nuclear fission.

The sun and other stars obtain its energy through the fusion of Hydrogen nuclei, in a reaction scheme known as the p-p cycle:

1. Deuterium Formation : ${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^2_1\text{e} + \nu$
2. Helium-3 Formation : ${}^2_1\text{H} + {}^1_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$
3. Helium-4 Formation : ${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2{}^1_1\text{H}$
4. Electron Annihilation : ${}^0_1\text{e} + {}^0_{-1}\text{e} \rightarrow 2\gamma$

Overall Reaction : ${}^1_1\text{H} + {}^0_{-1}\text{e} \rightarrow {}^4_2\text{He}$

Definition 20.16: Neutrino ν

A Neutrino ν is a neutral, very small particle of negligible mass (many orders of magnitude less than electrons).

Neutrinos are added to the nuclear reactions to account for various inconsistencies in conservation of momentum and nucleon properties such as spin and lepton number.

Though nuclear fusion produces extreme amounts of energy, has a large reserve of source fuel (hydrogen in water) and is much less hazardous than handling heavy nuclei, nuclear fusion reactions require temperatures up to 1×10^8 K to occur at reasonable rates and is hence unfeasible to be currently used as means of energy production.

20.5 Radioactivity

Definition 20.17: Radioactivity

Radioactivity is the random spontaneous decay of nucleus to a more stable nucleus, typically involving the emission of α, β and/or γ particles.

Definition 20.18: Spontaneity

A process is spontaneous if it is not affected by external factors such as temperature or pressure.

Definition 20.19: Randomness

A process is random if it cannot be predicted.

As for radioactivity, whether a nucleus decays within a time interval cannot be predicted. Nuclides of a single species are assumed to have a constant probability of decaying in a fixed period of time.

20.5.1 Radioactive Decay

Definition 20.20: α-decay

α-decay is the emission of a $^4_2$Hehelium nucleus (no electrons).

Definition 20.21: β-decay

β-decay is the emission of $^0_-$1eelectrons.).

Definition 20.22: γ-decay

γ-decay is the emission of high-energy photons.

Definition 20.23: Ionizing Power

The Ionizing Power of a radioactive decay is the ability of the decay particle to remove electrons from other external atoms. Ionizing power decreases from α to γ decay.

Definition 20.24: Penetrating Power

The Penetrating Power of a radioactive decay is the ability of the decay particle to pass through material before being absorbed. Penetrating power increases from α to γ decay.

Which form of decay occurred in a reaction can be inferred through inspection of the atomic mass numbers and charges of specific nuclei.

20.6 Activity and Half-Life

Definition 20.25: Law of Radioactive Decay

The Law of Radioactive Decay states that the rate of decay of a source of radioactive nuclei per unit time is proportional to the total number of nuclei present.

Equation 20.3: Law of Radioactive Decay

For a sample of N radioactive nuclei with decay constant N, the rate of decay $\frac{dN}{dt}$ is given by the equation

$$\frac{dN}{dt} = -\lambda N$$

Definition 20.26: Decay Constant λ

The Decay Constant λ of a nuclide is the probability that a nucleus will decay within a specific duration and has units Bq

Definition 20.27: Activity

The Activity of a radioactive source is the number of nuclear decays that occur per unit time in a source.

Definition 20.28: Becquerel

1 Bq is the progression of one radioactive decay reaction per second.

Definition 20.29: Half-life

The Half-life of a radioactive source is the time taken for its population to decrease to half of its previous value.

Equation 20.4: Half-life

A radioactive nuclei with decay constant λ and half-life $t_{1/2}$ are related by the equation

$$t_{1/2} = \frac{\ln(2)}{\lambda}$$

Equation 20.5: Equations for Radioactive Decay

For a radioactive source of decay constant λ and initial population N_0; its population N, activity A at time t are given by the equations

$$N = N_0 e^{-\lambda t} = N_0 \frac{1}{2}^{-t/t_{1/2}}$$

$$A = -\frac{dN}{dt} = \lambda N = A_0 e^{-\lambda t} = A_0 \frac{1}{2}^{-t/t_{1/2}}$$

20.6.1 Background Radiation

Definition 20.30: Background Radiation

Background Radiation is the ambient radiation in environment which people are exposed to.

The largest natural source of background radiation is the presence of airborne Radon $_{86}$Rn, among other sources such as cosmic rays and radiation from ^{14}C. The largest artificial source of background radiation comes from medical imaging equipment such as X-ray and CT scan machines.

When measuring the radiation given off by a sample of radioactive matter, the background radiation must be first measured and then used to treat the data to account for extra counts from background radiation.

20.6.2 Carbon Dating

^{14}C is a radioactive isotope of Carbon which has a half-life of 5730 years. Natural carbon samples contain 1.30×10^{-12} part C-14 due to nuclear reactions in the upper atmosphere.

Living organisms will maintain this ratio of C-14 to C-12 as carbon is constantly in exchange with the environment. However, once a organism dies its C-14 begins to decay without being replenished. By measuring the radioactivity of a sample of dead matter, the ratio of C-14 to C-12 can be established and hence the duration between its death and now can be calculated.