essential formulae.

Yes, U Can!

AREA OF ○ : πr^2

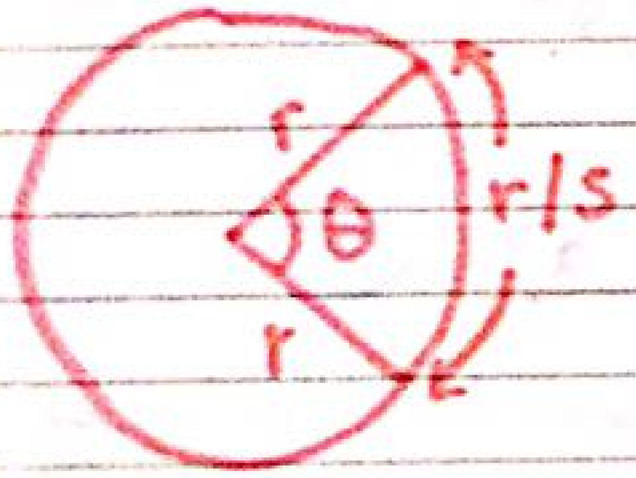
CIRCUMFERENCE : $2\pi r$

AREA OF SECTOR : DEG : $\frac{\theta^\circ}{360^\circ} \times \pi r^2$

AREA OF SECTOR : RAD : $\theta \times \frac{1}{2} r^2$

ARCLENGTH : DEG : $\frac{\theta^\circ}{360^\circ} \times 2\pi r$

ARCLENGTH : RAD : $r\theta$



If $r=1$, $\theta=1$

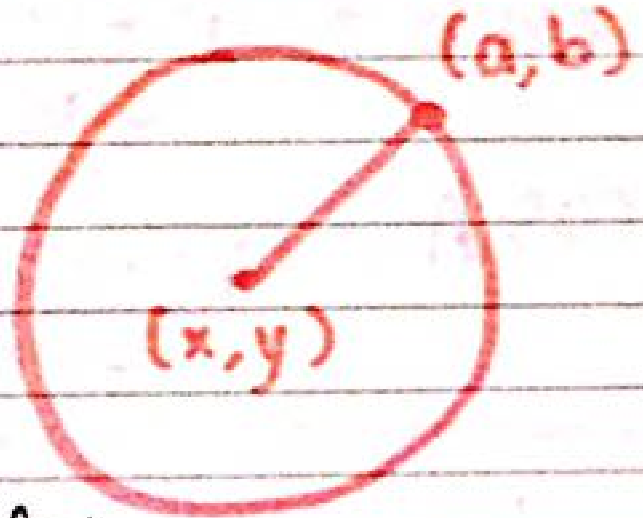


fig 1.

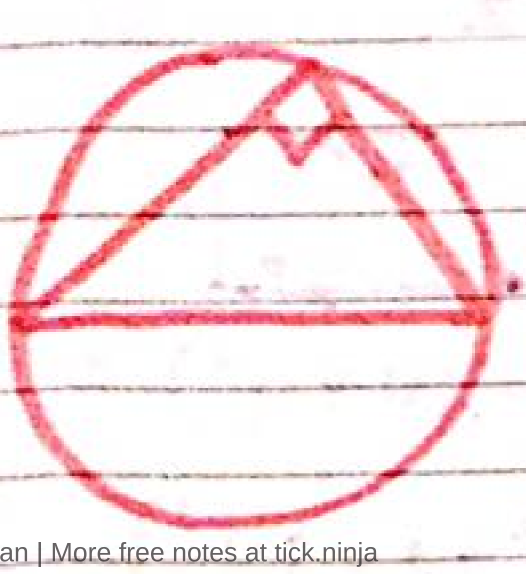
fig 2.

EQN OF ○ (fig 1.) : $(a-x)^2 + (b-y)^2 = r^2$

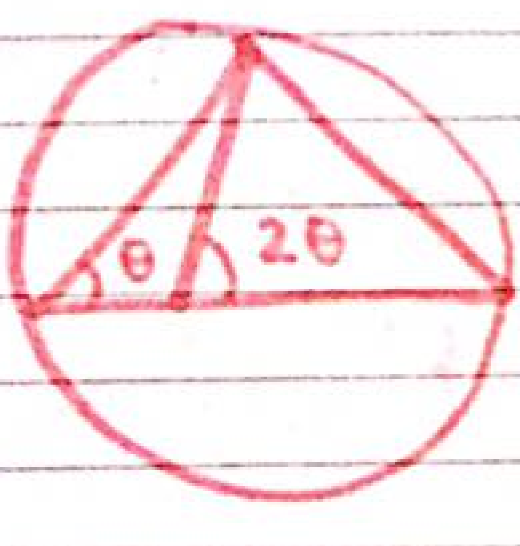
AREA OF SEGMENT (fig 2.) : sector − △

AREA OF △ : $\frac{1}{2} r^2 \sin\theta$ / $\frac{1}{2} ab\sin\theta$ / $\frac{1}{2} bh$

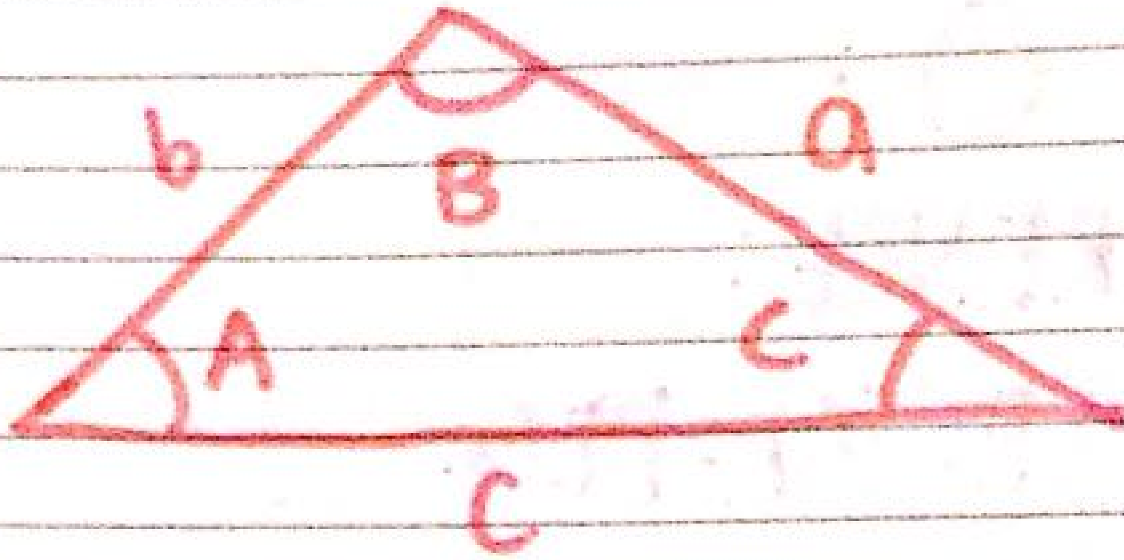
$a^2 + b^2 = c^2$ (pythag)



← rules →



essential formulae.



Sin rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cos rule: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Cone vol: $\frac{1}{3}\pi r^2 h$

Sphere vol: $\frac{4}{3}\pi r^3$

Cylinder vol: $\pi r^2 h$

Pyramid vol: $\frac{1}{3}h$ (base area)

Curved surface of cone: πr (slant side)

Curved surface of sphere: $4\pi r^2$

Trapezium area: $\frac{1}{2}h(a+b)$

Polynomial: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

complete square + THE FORMULA.

$y = ax^2 \pm bx + c$

$= a(x^2 \pm \frac{b}{a}x + \frac{c}{a})$

$= a(x^2 \pm \frac{b}{a}x + [\frac{b}{2}]^2 - [\frac{b}{2}]^2 + \frac{c}{a})$ ~~can skip this~~

$= a(x \pm \frac{b}{2})^2 - a(\frac{b}{2})^2 + c$

= complete! form = $y = a(x-k)^2 + h$

(k,h) = max/min pt

$y = mx + c$

$y - y_1 = m(x - x_1)$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

eg. Complete sq

$y = 2x^2 - 24x + 65$

$= 2(x^2 - 12x + \frac{65}{2})$

$= 2[(x-6)^2 - 36 + \frac{65}{2}]$

$= 2[(x-6)^2 - \frac{7}{2}]$

$= 2(x-6)^2 - 7$

laws: indices.

Yes,

$a^0 = 1$

$a^1 = a$

$a^{-m} = \frac{1}{a^m}$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{m/n} = \sqrt[n]{a^m}$

$(ab)^n = a^n b^n$

$(\frac{a}{b})^n = \frac{a^n}{b^2}$

$f(x) = a^x, \ x \in \mathbb{R}, \ a < 0, \ a \neq 1$

$a^x = a^n \Leftrightarrow x = n \Rightarrow a \neq 1, 0, -1$

- works with x-vals too!

laws of expo-log.

$y = \log_a x \iff a^y = x$

$\log_a(mn) = \log_a m + \log_a n$

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a(m^n) = n\log_a m$

$\log_a M = \log_a N \iff M = N$

$\log_a b = \frac{\log_c b}{\log_c a}$

$\frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$

eqn of tgt @ xpt for curve f(x)

STEP ONE.

Differentiate curve.

STEP TWO.

Subst x-pt into $\frac{dy}{dx}$.

This obtains: gradient at x-point.

STEP THREE.

Subst x-pt into curve equation.

This obtains: y-coordinate.

STEP FOUR.

$$y - y_1 = m(x - x_1)$$

↓ step3 ↓ step2 ↓ original x-pt

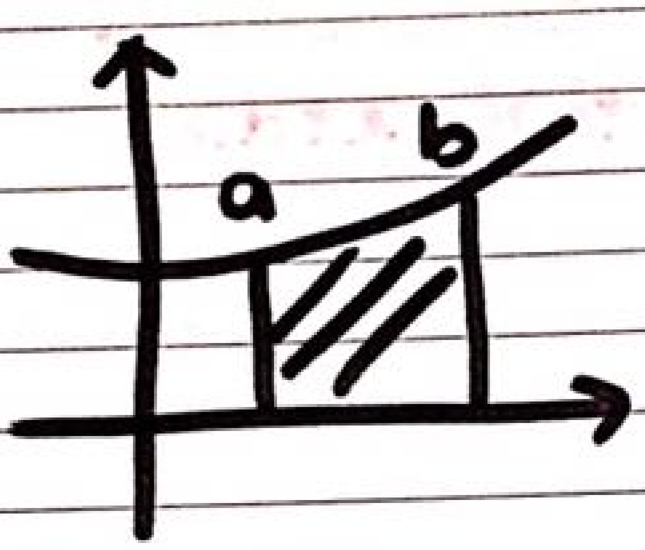
NORMAL?

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

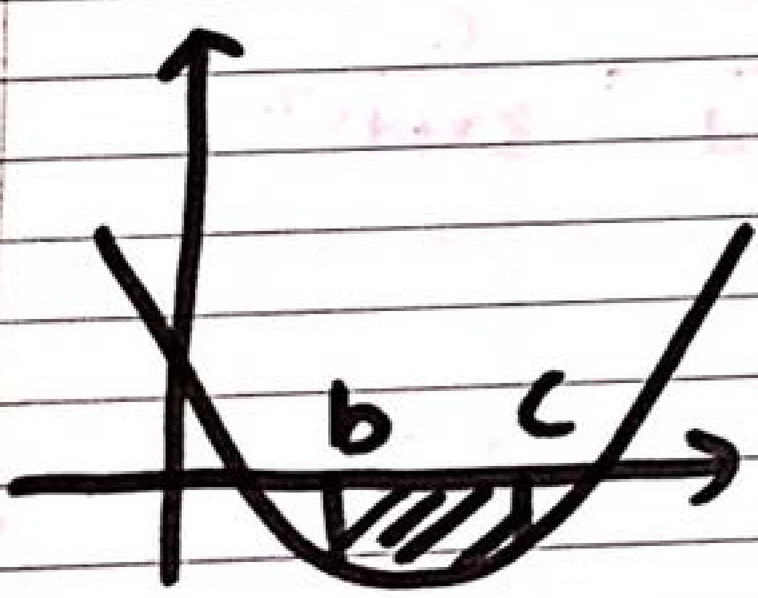
integrals.

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a$$

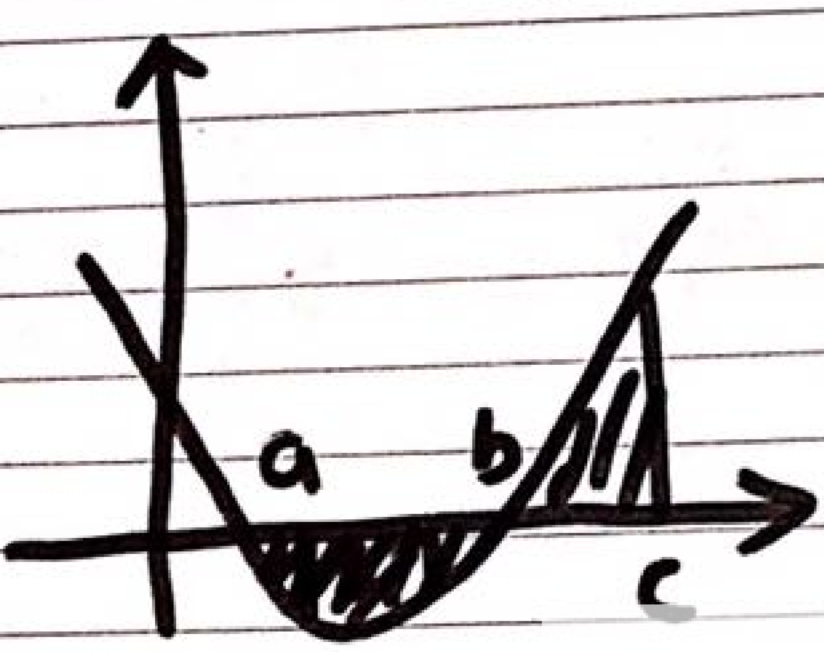
a = lower limit
b = upper limit
$f(x)$ = integrand
$F(b) - F(a)$ = integral



$$\int_a^b y\,dx$$



$$\left| \int_b^c y\,dx \right|$$



$$\int_a^b y\,dx + \left| \int_b^c y \right.$$

to find the derivative of a graph at $x = n$,
draw graph
CALC - 6 enter (n)

$f(x)$	$f'(x)$
x	1
x^2	$2x$
x^3	$3x^2$
-4	0
$x^{-1} / \frac{1}{x}$	$-x^{-2}$
$\sqrt{x} / x^{\frac{1}{2}}$	$\frac{1}{2}x^{-\frac{1}{2}}$
n^x	$n^x \ln(n)$
x^n	nx^{n-1}
$kf(x)$	$kf'(x)$
$n^{f(x)}$	$a^{f(x)} \cdot f'(x) \cdot \ln a$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) - g(x)$	$f'(x) - g'(x)$
$[f(x)]^n$	$n[f(x)]^{n-1} \cdot f'(x)$

$f(x)$	$f'(x)$
$\log_a x$	$\frac{1}{x\ln a}$
$\log_a[f(x)]$	$\frac{f'(x)}{f(x)\cdot(\ln a)}$
$\ln x$	$\frac{1}{x}$
$\ln ax$	$\frac{1}{x}$
$\ln[f(x)]$	$\frac{f'(x)}{f(x)}$
$\ln[f(x)]^n$	$n\cdot\frac{f'(x)}{f(x)}$
e^x	e^x
e^{ax}	ae^{ax}
e^{ax+b}	ae^{ax+b}
$e^{f(x)}$	$e^{f(x)}\cdot f'(x)$

$\int f(x)\,dx$	$= F(x) + C$
ax^n	$\frac{ax^{n+1}}{n+1} + C$
$k \cdot f(x)$	$k \cdot F(x) + C$
$f(x) + g(x)$	$F(x) + G(x) + C$
$f(x) - g(x)$	$F(x) - G(x) + C$
k	$kx + C$
kx^n	$\frac{kx^{n+1}}{n+1} + C$
$k(ax+b)^n$	$\frac{k(ax+b)^{n+1}}{a(n+1)} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\frac{1}{ax+b}$	$\frac{1}{a}\ln\lvert ax+b\rvert + C$
$\ln x$	$x\ln(x) - x + C$
e^x	$e^x + C$
e^{-x}	$-e^{-x} + C$
e^{ax+b}	$\frac{1}{a}e^{ax+b} + C$
k^{ax}	$\frac{1}{a\ln k}k^x + C$

inequalities & roots

Yes, U Can!

> : larger than

< : smaller than

≤ ≥ : or equal to

If c +ve

$a<b$, $ac<bc$

If c -ve

$a<b$, $ac>bc$

$a<b$, $-a>-b$

$a<b$, $\frac{1}{a}>\frac{1}{b}$

$-a > -b$ $\quad$ $a > b$

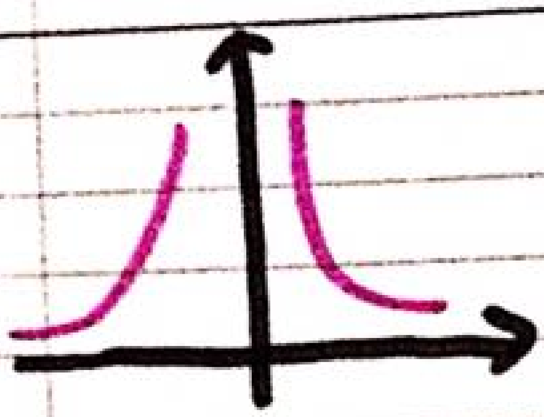
$(-a)^2 < (-b)^2$ $\quad$ $a^2 > b^2$

multiply both sides by sq of denom to clear w/o changing signs.

$D = b^2 - 4ac$

- >0 2 real & distinct
- $=0$ 2 real & equal
- <0 no real
- ≥ 0 real roots

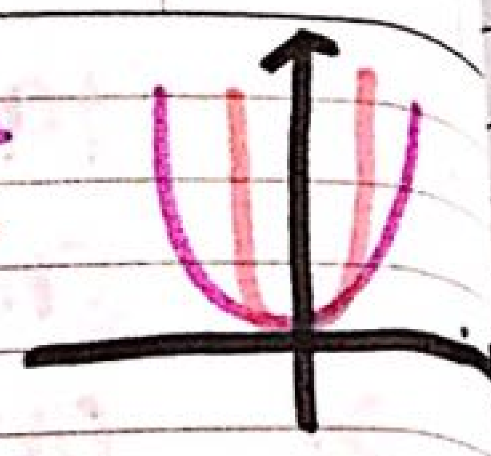
graph shapes/asymptotes

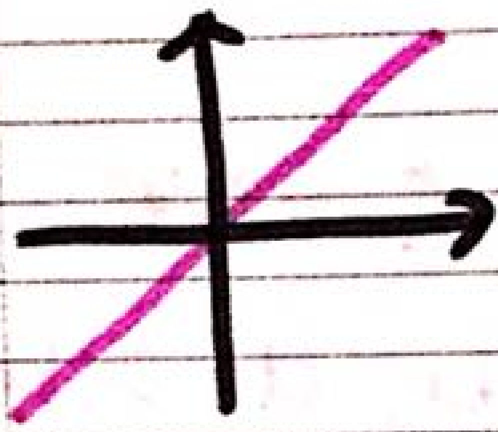


$y = x^{-2}$

$y = x^2$

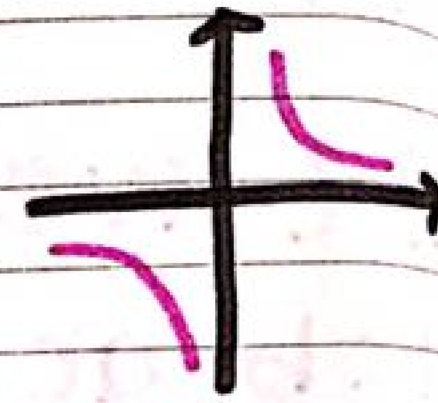
$y = x^4$

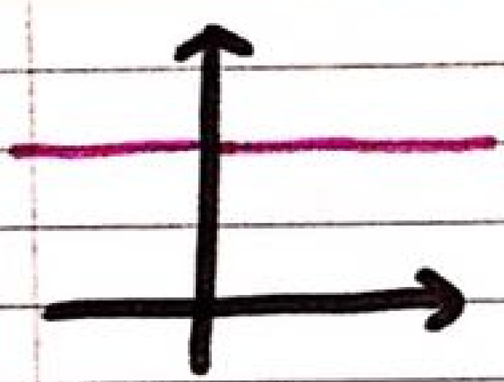




$y = x^1$

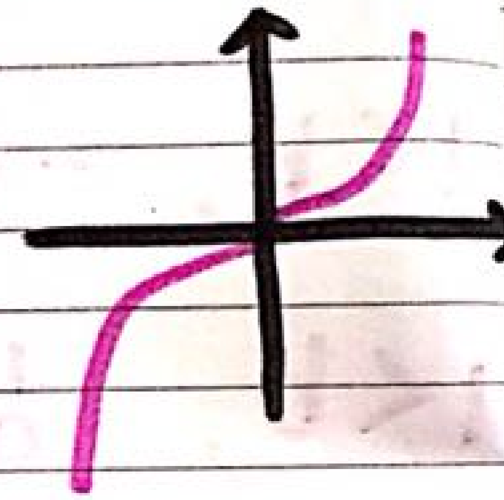
$y = x^{-1}$

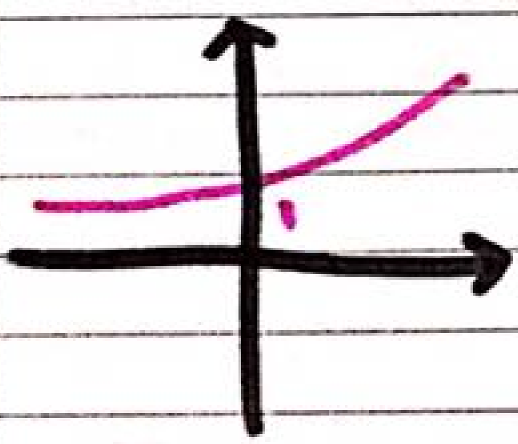




$y = x^0$

$y = x^3$





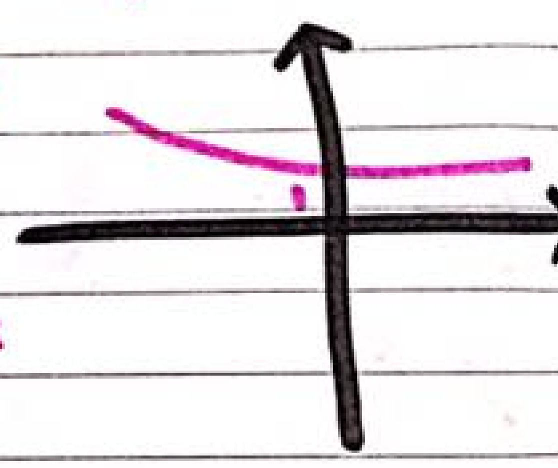
$y = a^x$

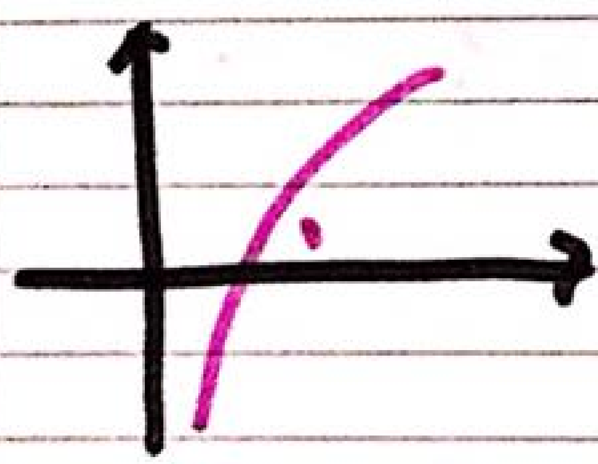
$a > 1$

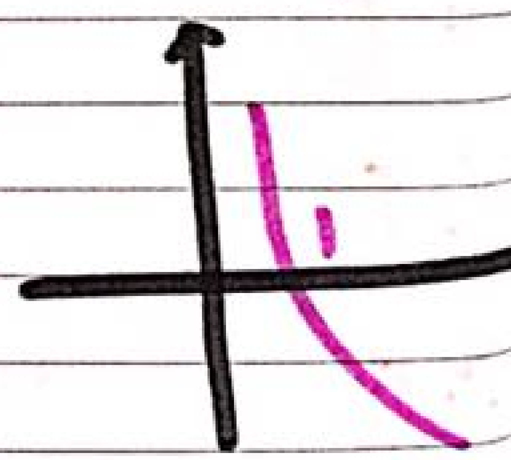
$y = a^x$

$0 < a < 1$

$y = a^{-x}$







$\log_a x$

$a > 1$

$\log_a x$ / $-\log_a x$

$0 < a < 1$

Whatever value y=? cannot take is an asymptote.

eg. $y = 1 - \frac{1}{x+1}$

× 1   x −1

eg. $y = 1 + \frac{3}{x-2}$

(1) → y-asymp   (x) → x-asymp

ASYMPTOTES

$e^x \rightarrow y = 2 - e^{x-2}$

$\ln x \rightarrow y = 1 + \ln(x+3)$

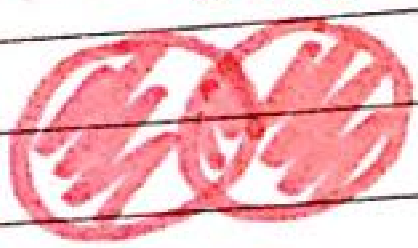
↓

solve for 0

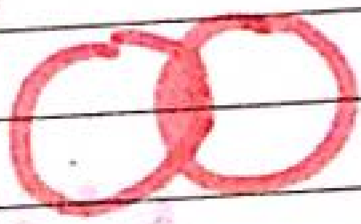
PROBABILITY.

$P(\text{event}) = \dfrac{\text{number of outcomes favourable to occurence}}{\text{total number of outcomes}}$

$P(A \cup B)$ — ~~[illegible]~~



$P(A \cap B)$ — intercept



$P(A \cap B) = 0$ — mutually exclusive

COMPLEMENTARY RULE

Complement = set of all outcomes in sample space that are NOT the

$P(A \cap A') = 0$ $P(A \cup A') = 1$

GLOSSARY

$\mathcal{E}$	universal set
$\in$	element of
$\subset$	subset of
Ω	sample space
Σ	all x / all value
$\forall$	for all
$\mathbb{R}$	real
$\setminus\{n\}$	all except n

probability

Yes, U

$P(A) + P(B) = P(A \cup B)$

if mutually exclusive

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

if not

CONDITIONAL PROBABILITY

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

INDEPENDENT EVENTS

$P(A|B) = P(A)$ / $P(B|A) = P(B)$

$P(A \cap B) = P(A) \times P(B)$

A & B = indie

AT LEAST

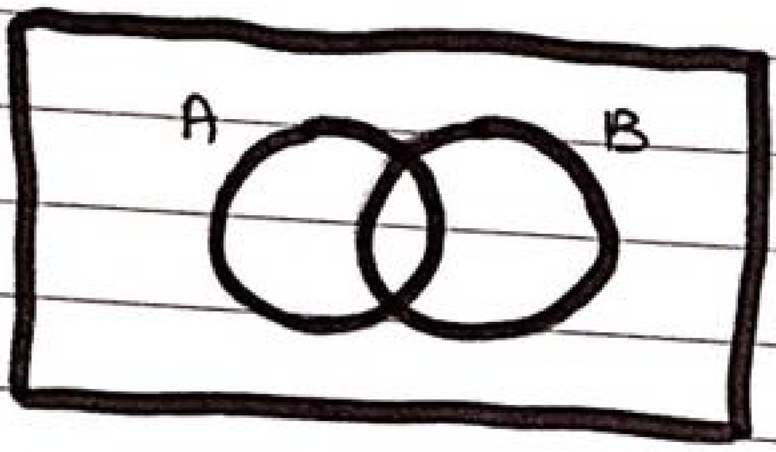
$P(A') = 1 - P(A)$

probability diagrams

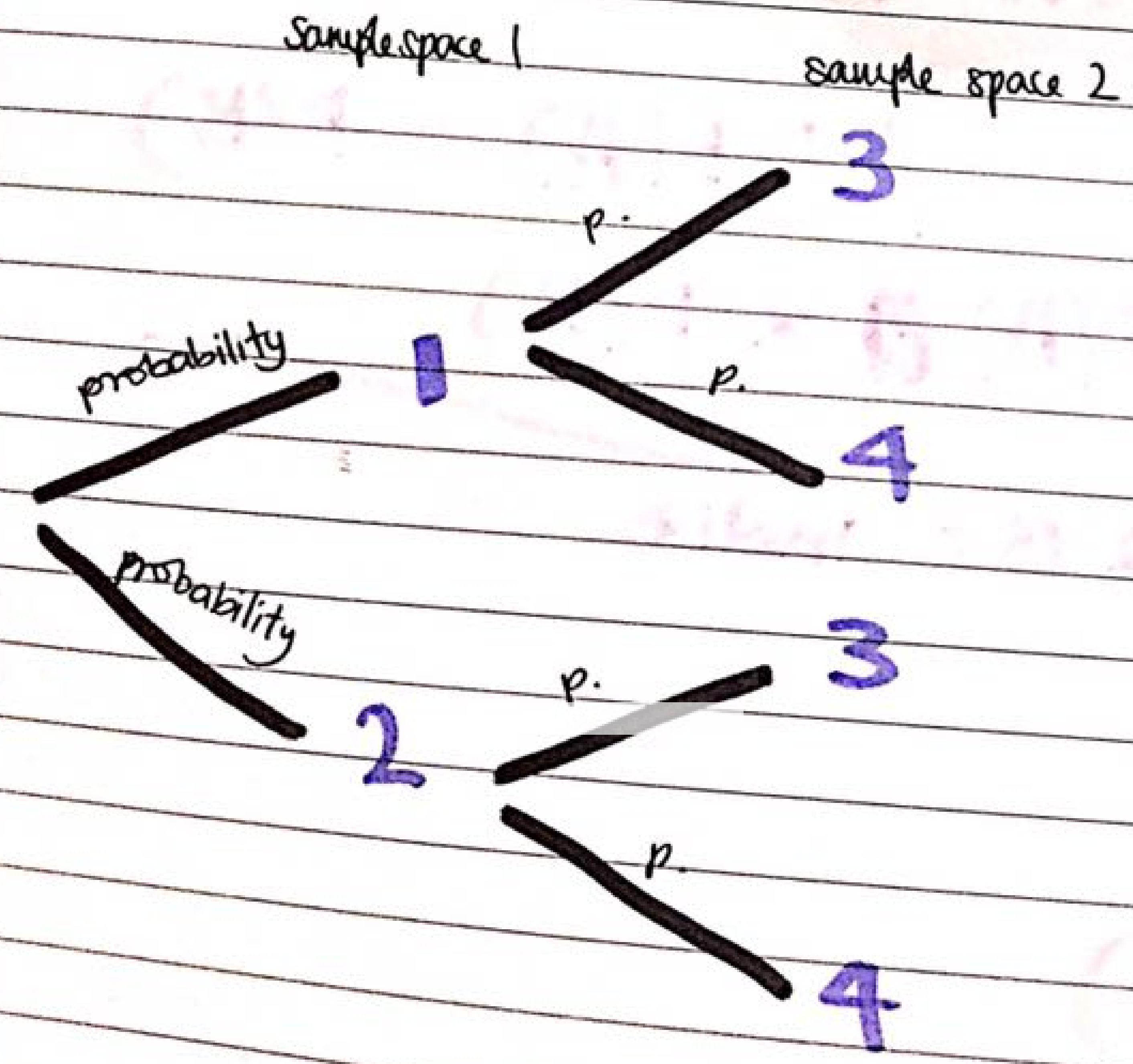
possibility.

	1st toss	2nd toss
head	H1	H2
tail	T1	T2

venn.



tree.



random variable + binom distr

AN EXAMPLE:

3 coins are tossed. X to denote no. of heads obtained. X will take values 0, 1, 2, 3. AKA result of random xperiment.

$P(X = x)$

probability that random variable X yields value x.

$\therefore P(X=2) \rightarrow$ probability that ~~need~~ we get 2 heads...

$$\sum_{\text{all } x} P(X=x) = 1$$

$X \sim B(n, p)$

- X: total no. of outcomes obtained
- n: no. of repeated trials
- p: probability of success for each trial

$$P(X=x) = \binom{n}{x} \times p^x \cdot (1-p)^{n-x}$$

---

binompdf: $P(X=x)$

binomcdf: $P(X \leq x)$

mean: $E(X) = np$

variance: $Var(X) = np(1-p)$

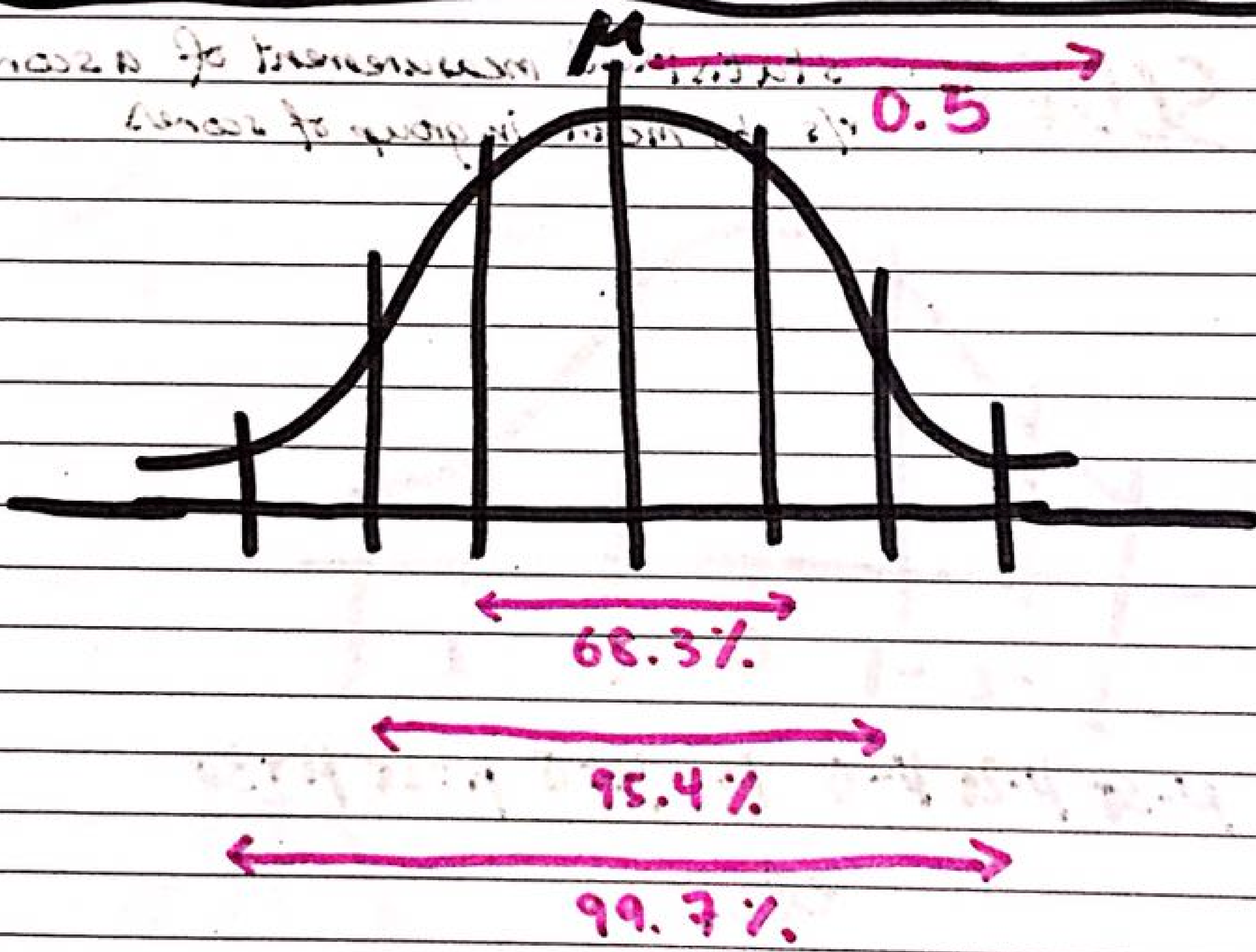
[redacted] norm distr.

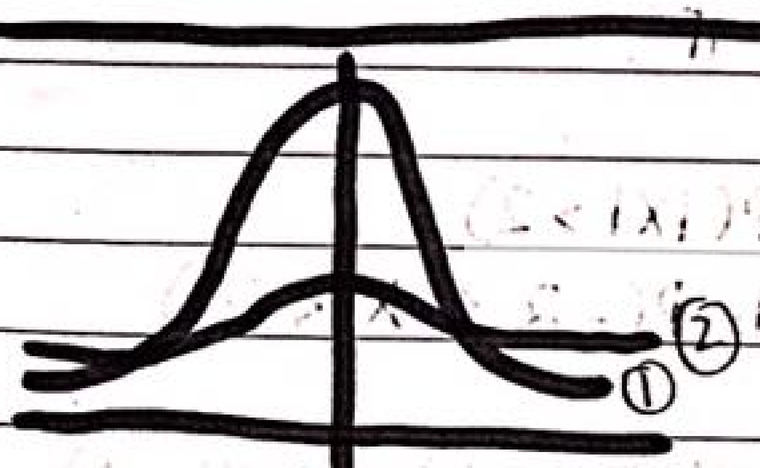
$E(X) = \mu$ — MEAN

$Var(X) = \sigma^2$ — VARIANCE

STD DEV = σ

$X \sim N(\mu, \sigma^2)$





$\mu_1 = \mu_2$
$\sigma_1 < \sigma_2$

$\mu_1 < \mu_2$
$\sigma_1 = \sigma_2$

STD NORM DISTR

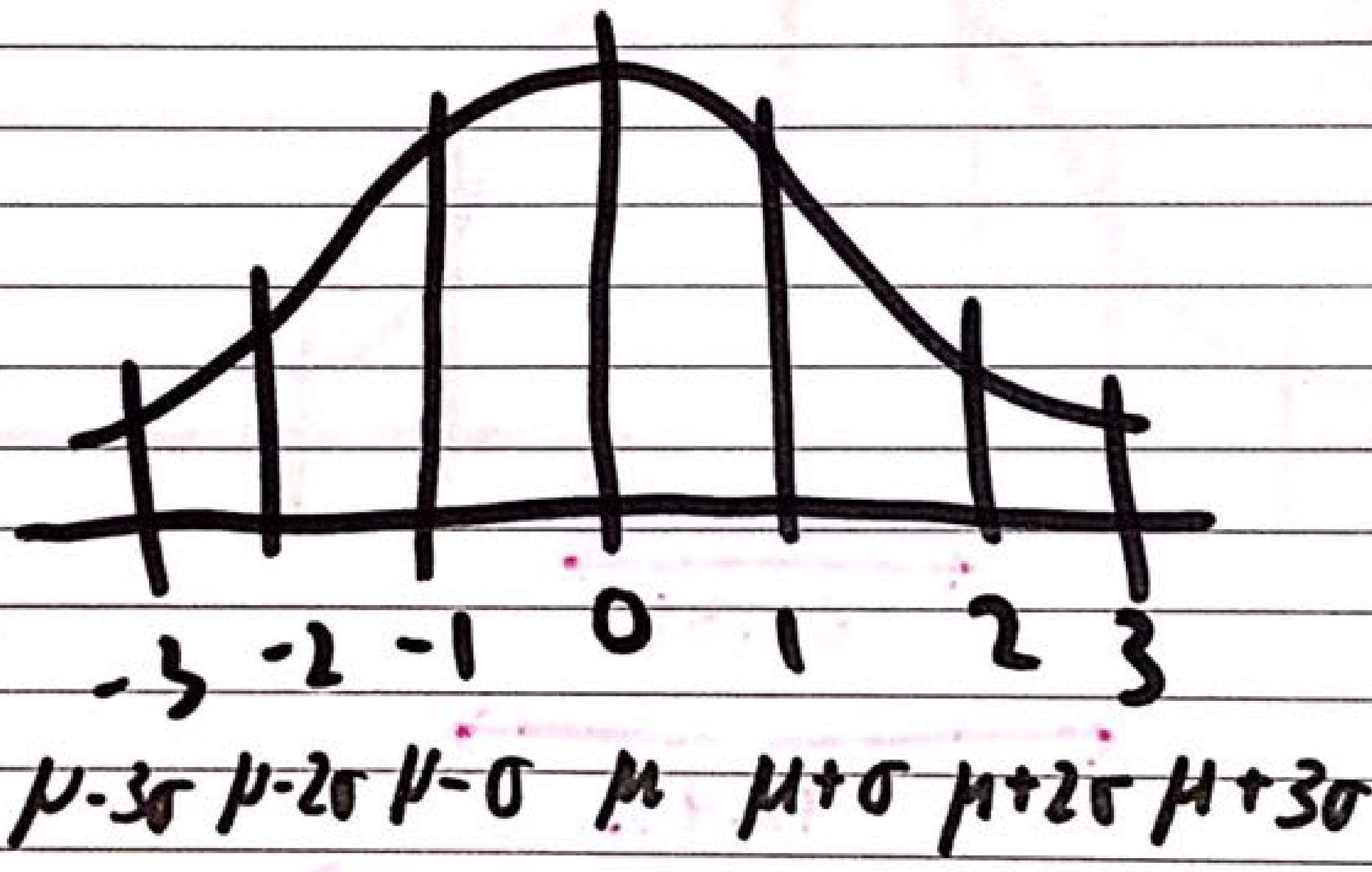
Date

$Z \sim N(0, 1)$

$$Z = \frac{X - \mu}{\sigma}$$

value you wish to find Z score for
mean
standard dev.

Z SCORE: statistical measurement of a score's r/s to mean in group of scores



etc.

IF

$P(X < n)$ to find σ,

$P(Z < \frac{X-\mu}{\sigma}) = \text{area}$

INVNORM area, 0, 1

$\frac{X-\mu}{\sigma} = \text{answer}$

IF

$P(|X| > 2)$,

$1 - P(-2 < X < 2)$

$P(|X| < 1)$,

$P(-1 < X < 1)$

NORM DISTR [redacted]

NORMALCDF

$P(X < A)$

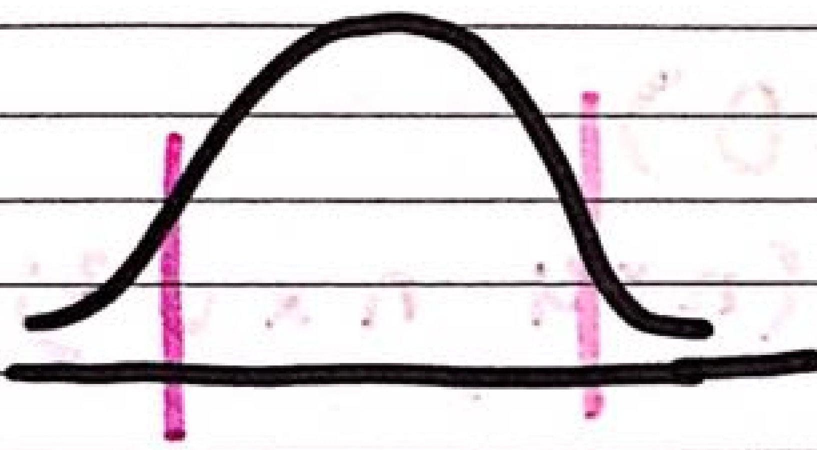
upper limit: A
lower limit: - E99

$P(X > A)$

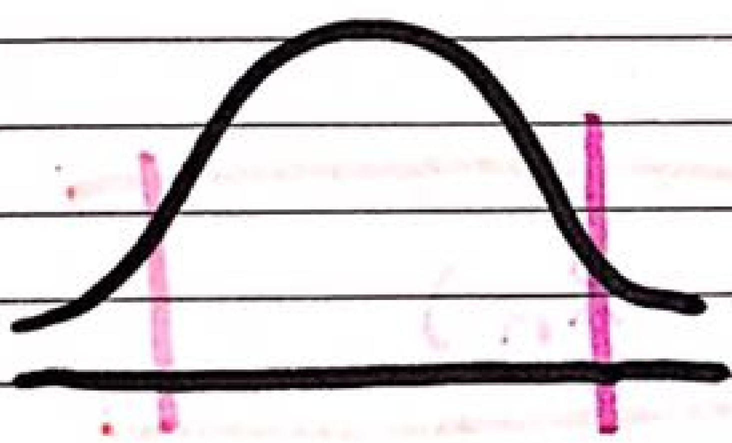
upper limit: E99
lower limit: A

INVNORM

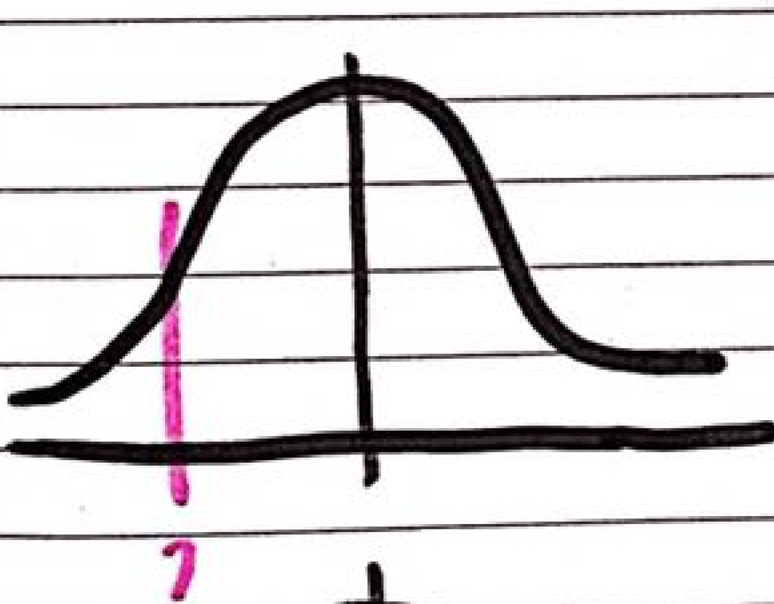
large no. | small no.
calculates area to left



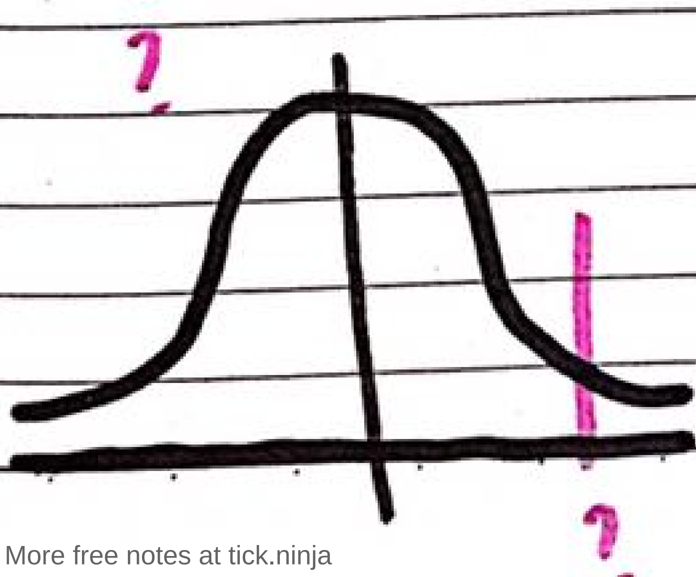
$P(X \leq ?) = n$
n



$P(X \geq ?) = n$
n



$P(? \leq X \leq \mu) = n$
n



$P(? \geq X \geq \mu) = n$
n

LINEAR NORM

Date

$X+Y \sim N(+,+)$

$X-Y \sim N(-,+)$

$nX \sim N(n \times \mu, n^2 \times \sigma^2)$

$|X+Y| < n \approx P(-n < X+Y < n)$

$P(X<Y) = P(X-Y<0)$

$X_1 + X_2 + \dots X_n \sim N(n \times \mu, n \times \sigma^2)$

$\bar{X} \sim (\mu, \frac{\sigma^2}{n})$

$\bar{X} = \frac{1}{n}(X_1 + X_2 + \dots X_n)$

getting everything with raw data.

> STAT

> CALC, 1 (1-VAR-STAT)

> List: L1, FreqList: L2

> Calculate

> Sx : unbiased est of var.

~~(Sx)²/~~

> the bigger no. is UEoV.

CONTINUITY CORRECTION

Date No

$X \sim B(n, p)$:

Since n= ? is large,

np = ① > 5, and

n(1-p) = ② > 5,

$X \sim N(①, ②)$ approx.

∴ P(X □ ____) = P(X □ ____) by CC

what you're trying to find eg. $X \geq 0.6$ = ans.

SCENARIOS

$P(X > 5) = P(X > 5.5)$

$P(X < 5) = P(X < 4.5)$

$P(X \geq 5) = P(X \geq 4.5)$

$P(X \leq 5) = P(X \leq 5.5)$

CENTRAL LIMIT THEOREM

Date:

USE WHEN: samples taken from distribution not norm distr. AKA no $N \sim (np, np(1-p))$.

FIND:

n. sample size.

μ. $E(X)$ / np / mean

σ^2. $Var(X)$ / $np(1-p)$ / variance

PRESENT MEAN:

Since n = ? is large, by CLT,

$\overline{X} = \frac{1}{n}(X_1 + X_2 + \dots X_n) \sim N(\mu, \frac{\sigma^2}{n})$

PRESENT SUM:

Since n = ? is large, by CLT,

$S = X_1 + X_2 + \dots X_n \sim N(n\mu, n\sigma^2)$

$\Sigma(x \pm n) = ?$

↓

$\bar{x} = \frac{\Sigma(x \pm n)}{a} \mp n$

aka

$\Sigma(x-2) = 5$

↓

$\bar{x} = \frac{5}{20} + 2$

UNBIASED MEAN

> 2nd STAT (LIST)
> MATH 3 (mean)
> mean (L1)

$\overline{x}$ ↓ $\Sigma x / n$

UNBIASED VAR

> 2nd STAT (LIST)
> MATH 8 (variance)
> var (L1)

s^2 ↓ $(1/n-1)[\Sigma x^2 - (\Sigma x)^2/n]$

> STAT
> Edit, 1
> key list of numbers (L1)

TO SORT ASCD

> STAT
> Sort A
> Sort A (L1)
> STAT
> EDIT, 1

SAMPLING METHODS

1. RANDOM.

def. each member of pop^n = equal chance of being selected.

adv. data collected free from bias, easy analysis
disadv. if pop^n large, hard to identify every member, possible inaccessability

how. generate ordered list after assigning numbers, random generator

2. SYSTEMATIC.

def. select members at regular intervals.

pro. evenly spread out, easy to conduct
con. if cyclical pattern, bias, not always possible to line up.

how. randomly select starting point, go.

3. STRATIFIED.

def. representative non-overlapping strata, proportional sample size.

pro. more likely to be representative, can analyse strata separately.
con. difficult to identify strata, time-consuming.
eg. need 10. 10 boys 90 girls - pick 1 boy 9 girls. done.

4. QUOTA.

def. non-probability. mutually xclusive strata, unproportional sample size.

pro. faster, low cost
con. bias, not representative of pop^n
eg. need 10. 10 boys 90 girls - pick 5 boys 5 girls. done.

HYPOTHESIS TESTING

Date

		pop^n NORMAL	pop^n UNKNOWN
pop^n var/σ^2 KNOWN	$n \geq 50$ (large)	$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$	$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$ by CLT, approx
	$n < 50$ (small)	$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$	assume X normal $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$
pop^n var/σ^2 UNKNOWN s^2 used	$n \geq 50$	$\bar{x} \sim N(\mu, \frac{s^2}{n})$ approx	$\bar{x} \sim N(\mu, \frac{s^2}{n})$ by CLT, approx
	$n \leq 50$	NOT IN SYLLABUS	

> STAT
> TESTS
> Z-TEST
> STATS
> key in stuff
> choose arrow according to question

<	decrease	} 1 tail
>	increase	
≠	difference	} 2 tail

WHAT DOES __% LOS MEAN?

There is a probability of ___ (in decimals) of rejecting claim that ______ (fill in w/ context of q^n) is ____ (given mean) when it is actually true.

HYPO QUESTION TYPES

Date No.

TYPE 1.

mean provided, σ^2/s^2 provided, test for probability @ __ % LOS.

let X (in units) denote ______ (what's measured) & let $E(X) = \mu$

$H_0: \mu =$ given mean
$H_1: \mu \neq$ or $\leq$ or $\geq$ given mean

We perform a 1/2-tail test at __ % LOS.

Under Ho, $\bar{X} \sim N(\quad , \quad)$ → see table
$\bar{x}$ = tested mean
From GC, p = ___

p < / less than LOS in decimals (eg 5% = 0.05),
Ho rejected, sufficient evidence at __ % LOS that ______

p > / more than LOS,
Ho not rejected, insufficient evidence etc.

TYPE 2.

mean not provided, σ^2/s^2 provided, test @ __ % LOS, Ho reject / do not reject

For Ho reject, $P(\bar{X} < \bar{x}) <$ __ LOS (in dec)
For H_1 reject, $P(\bar{X} < \bar{x}) >$ __ LOS (in dec)

invnorm LOS in dec, μ, σ.

HYPO QUESTION TYPES

Date

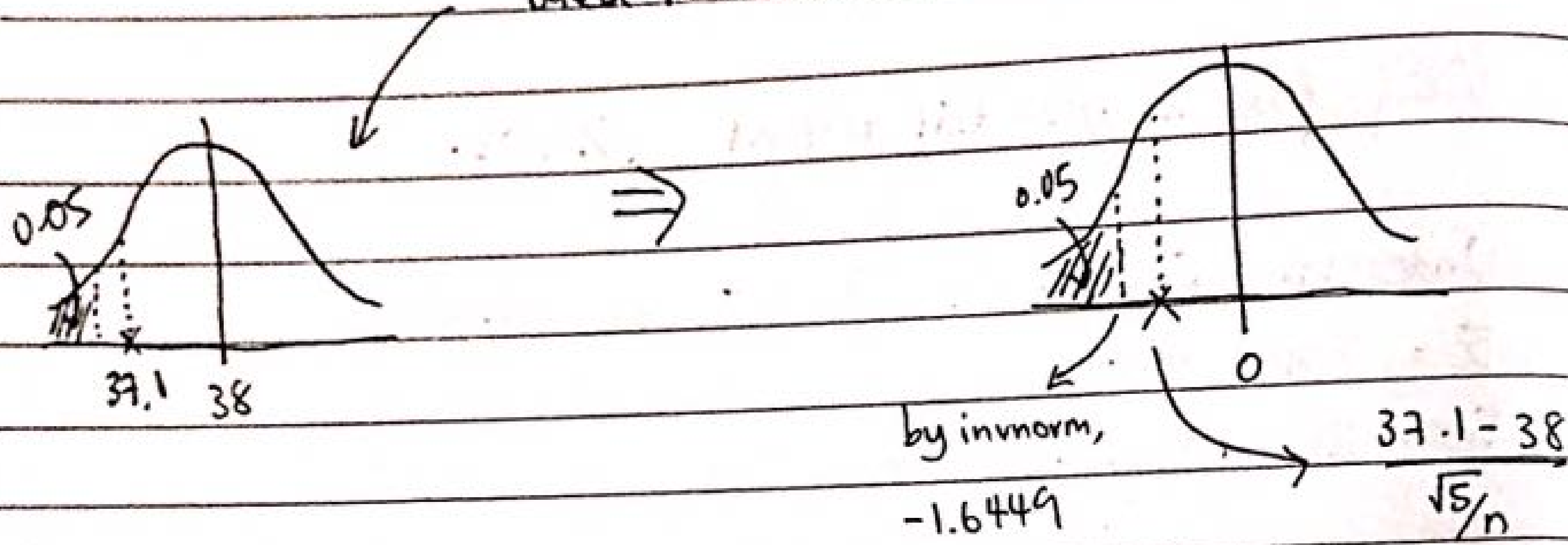
TYPE 3.

mean provided, σ^2/s^2 provided, test@ _%LOS, sample size NOT provided, H_0 do not r…

Use Z-SCORE.

eg... ~~if~~ if $P(\bar{X} < 37.1) > 0.05$, $\bar{X} \sim N(38, \frac{5^2}{n})$

area has to be > 0.05.



$\therefore \frac{37.1-38}{\sqrt{5}/n} > -1.6449$ (solve arith)

TYPE 4

tested mean/σ^2/s^2 provided, test@ _%LOS, given mean NOT provided, H_0 do not/reject

$H_0: \mu = \mu_0$

$H_1: \mu \neq$ or $\geq$ or $\leq \mu_0$

If $<$, left side

If $>$, right side

$\bar{X} \sim N(\mu_0, \frac{\sigma^2/s^2}{n})$ ...

For H_0 reject, $P(\bar{X} > \bar{x}) <$ _ LOS index

For H_0 accept/do not reject, $P(\bar{X} > \bar{x}) >$ _ LOS index

See Type 3 diagram. Standardise as necessary. Arithmetic.

CORRELATION

DRAW THE SCATTER PLOT

STAT > EDIT > x = L1 (input values in L1) > y = L2 (input values in L2) >
2nd > y = / STATPLOT > 1, ON, [:·:], + > ZOOM 9

FIND r / CORRELATION COEFFICIENT

STAT > EDIT > x = L1 > y = L2 > STAT > CALC > 8 (LINREG a + bx)
> CALCULATE

DRAW BEST FIT LINE (least □ regression line of y on x)

after ZOOM 9, > STAT > CALC > 8 > STORE REG EQ > ALPHA F4
> y₁ / whatever one you want > CALC > ZOOM 9.

If y ↗ as x ↗,	+ve / direct correlation	r = 1
If y ↘ as x ↗,	-ve / inverse correlation	r = -1
If no r/s,	uncorrelated	r = 0

HOW TO GET LEAST □ REGRESSION LINE OF Y ON X

See correlation coefficient but use values of a & b in eqn a + bx
If you know x & trying to find y, subst.
If you know y & trying to find x,
STAT > EDIT > x = L2 > y = L1 > STAT > CALC > 8 > CALCULATE!
Repeat.

RELIABILITY

1. STRONG CORRELATION (far from 0)
AND
2. WITHIN GIVEN DATA RANGE