Senior's Tuition And Mentorship Program 3.0 Year 2s

2M.2 Algebra 2 (Question types)

There are 4 fundamental question types in Algebra for Year 2.

1) **Solving simultaneous equations in second degree**
2) **Making a term the subject**
3) **Factorisation and simplification of an expression**
4) **Solving a multi-degree single variable equation**

Every question tested can be turned into one of the above question types.

1) **Solving simultaneous equations in second degree**

For year 2, there will be many simultaneous equations where one of the equations is a linear equation and the other is a second degree equation. The general technique of solving such equations should be understood and applied accordingly.

a. Making one of the variables the subject in the linear equation. The choice of variable should be made with your end-goal in mind: to make calculations as painless as possible
b. Substitute this value into your second equation
c. Solve the resultant quadratic equation by factorisation or quadratic formula

You want to choose the set of variables that gives you the least amount of calculations in the end so as to prevent making mistakes. For instance,

$$\begin{cases} 4x + 5 = 5(y + 2) \\ (y - 4)^2 = x(y - 1) - 9 \end{cases}$$

We first make either *x* or *y* the subject in the first equation (the linear one). But which one should we attack – the *x* or the *y*? The answer lies in the second equation. We see that the *x* term only appeared once and is by itself. This means that it will be much easier to substitute the resulting expression into the second equation.

Solution:

From first equation we have $x = \frac{5(y+1)}{4}$

Substituting this into the second equation we have $y^2 - 8y + 16 = \frac{5(y+1)(y-1)}{4} - 9$

Multiplying through by 4 we have $4y^2 - 32y + 64 = 5y^2 - 5 - 36$

Rearranging terms we have $y^2 + 32y - 105 = 0, \quad \therefore (y + 35)(y - 3) = 0\,, y = 3\ or\ -35$

When $y = 3, x = 5\times\frac{3+1}{4} = 5$, when $y = -35, x = 5\times\frac{-35+1}{4} = -\frac{85}{2}$

As can be seen, solving such equations is not really hard. But one has to watch out for multiple tricks and variations → almost every simultaneous equation, or a question that talks about how 2 graphs intersect at two points (and asks you to find the coordinates of these 2 points) can be solved with this method. What are some of the changes that teachers can make to these questions?

i) Change the variables to complicated terms: you need to use substitution for this. Sometimes this is not necessary but it will greatly aid in calculations.
ii) Change in question type: asking for the coordinates of points of intersection between two equations on a graph

For i) look out for similar terms that keep appearing in the equations, such as $\frac{1}{x}$, x^2, $(x+3)$, etc.

2) Making a term the subject

Not too much will be said about this point. But the main idea is, no matter how complicated the expression looks, always *try something* first. See square roots? Square both sides. See fractions? Cross multiply. These questions sometimes require you to explore a bit before realising that the solution is right in front of you.

3) Factorisation and simplification of an expression

The 2M.1 has already touchedon this point. I will not go into it too much, but take note of the following rules as revision:

a) Take out all common factors FIRST. Please do not forget this point.
b) When there are 2 terms, consider trying to make them in the form $x^2 - y^2$
c) When there are 3 terms, consider cross factorisation (i.e. quadratic factorisation)
d) When there are more than 3 terms, consider grouping.

4) Solving multi-degree single variable equations

a) Look for similar terms that you can factor out. If there is one term (x+1) for example that appears in every single part of the equation, factor it out. DO NOT divide both sides by a factor, as it might be a solution (what if x=-1, making x+1=0?)
b) If there are no similar terms, expand everything, cross multiply denominators and rearrange the terms
c) Try to factorise and solve the equation

Once again, the emphasis is on practice. With the above tips and rules to solving the different question types, please attempt as many problems as you can.