MATHEMATICS OMEGA SHEET

Rules of Indices

1. **(a)** $a^m \times a^n = a^{m+n}$

   (b) $a^m \div a^n = a^{m-n}$

   (c) $(a^m)^n = a^{mn}$

   (d) $a^0 = 1$, provided $a \neq 0$

   (e) $a^{-n} = \dfrac{1}{a^n}$

   (f) $a^{\frac{1}{n}} = \sqrt[n]{a}$

   (g) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

   (h) $(a \times b)^n = a^n \times b^n$

   (i) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$, provided $b \neq 0$

Definition of a Surd

2. A surd is an irrational root of a real number, e.g. $\sqrt{2}$, $\sqrt{3}$, etc..

Operations on Surds

3. **(a)** $\sqrt{a} \times \sqrt{a} = a$

   (b) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

   (c) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

   (d) $m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

   (e) $m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$

Conjugate Surds

4. $a\sqrt{m} + b\sqrt{n}$ and $a\sqrt{m} - b\sqrt{n}$ are conjugate surds.

5. $\left(a\sqrt{m} + b\sqrt{n}\right)\left(a\sqrt{m} - b\sqrt{n}\right) = a^2m - b^2n$, which is a rational number.

6. The product of a pair of conjugate surds is always a rational number.

Rationalising the Denominator

7. To rationalise the denominator of a surd is to make the denominator a rational number.

(a) $$\frac{\sqrt{b}}{\sqrt{a}} = \frac{\sqrt{b}}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$$
$$= \frac{\sqrt{ab}}{a}$$

(b) $$\frac{1}{\sqrt{a}+\sqrt{b}} = \frac{1}{\sqrt{a}+\sqrt{b}} \times \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$
$$= \frac{\sqrt{a}-\sqrt{b}}{a-b}$$

Logarithms

8. If $a > 0$ and $a \neq 1$,

index

exponential form $\longrightarrow$ $\boxed{y = a^x}$ is equivalent to $\boxed{x = \log_a y}$ $\longleftarrow$ logarithmic form

base

9. For $\log_a y$ to exist, a and y must be positive.

10. Common logarithm: $\log_{10} x$ or $\lg x$

11. Natural logarithm: $\log_e x$ or $\ln x$

Laws of Logarithms

12. Product Law: $\log_a xy = \log_a x + \log_a y$

13. Quotient Law: $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

14. Power Law: $\log_a x^r = r \log_a x$

Change of Base Formula

15. $$\log_a x = \frac{\log_b x}{\log_b a}$$

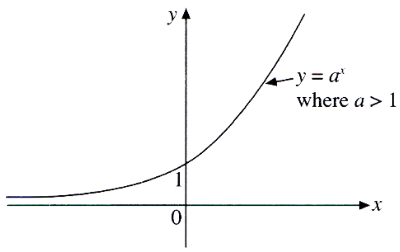
Other Properties of Logarithms

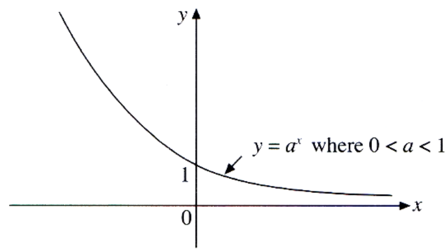
16. **(a)** $\log_a a = 1$

(b) $\log_a 1 = 0$

Graphs of Exponential Functions

17. Graphs of $y = a^x$

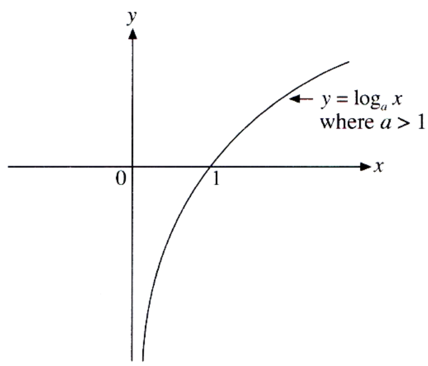


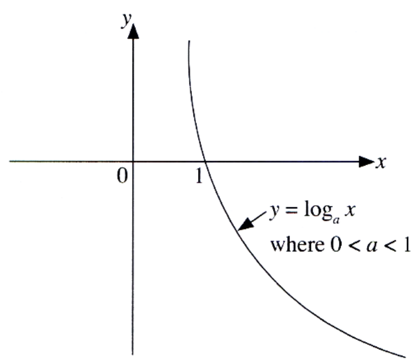


The graph of $y = a^x$ must pass through the point $(0, 1)$ because $a^0 = 1$.

Graphs of Logarithmic Functions

18. Graphs of $y = \log_a x$





Solving Exponential Equations

19. Given $a^x = b$,

- If b can be expressed as a power of a, e.g. $b = a^y$, then $a^x = a^y$ $\therefore x = y$.
- If b cannot be expressed as a power of a,
  - take common logarithms on both sides, i.e. $x \lg a = \lg b$ $\therefore$ $x = \dfrac{\lg b}{\lg a}$, or
  - take natural logarithms on both sides if $a = e$, i.e. $x \ln e = \ln b$ $\therefore x = \ln b$

20. Given $p\left(a^{2x}\right) + q\left(a^x\right) + r = 0$,

Step 1: Substitute $u = a^x$ to get a quadratic equation $pu^2 + qu + r = 0$.

Step 2: Solve for u and deduce the value(s) of x.

Solving Logarithmic Equations

21. To solve logarithmic equations,

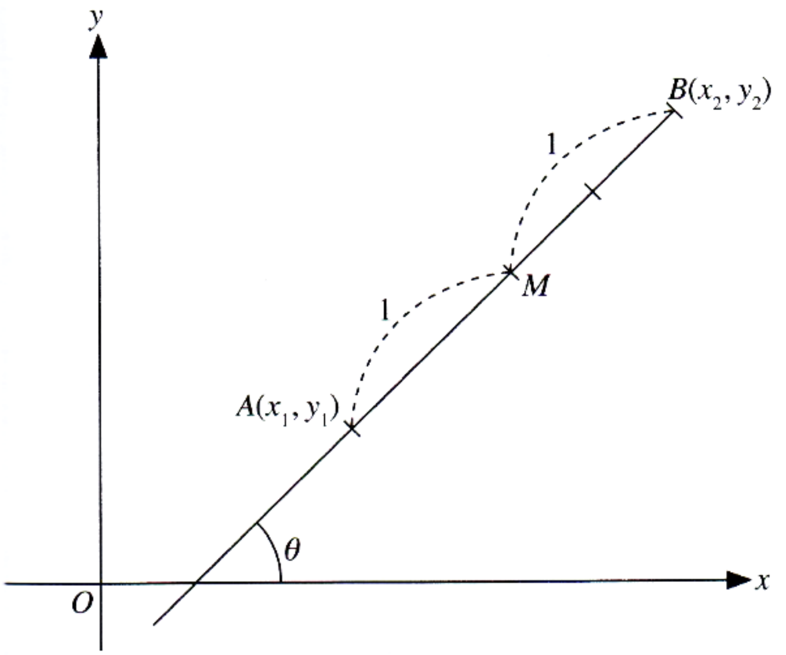
Step 1: Change the bases of the logarithmic functions to the same base. We usually choose the smaller base as the final base.

Step 2: Use one of the following methods to solve the equations.

(a) If $\log_a x = \log_a y$, then $x = y$ and vice versa.

(b) If $\log_a x = b$, then $x = a^b$.

(c) Use the laws of logarithms to combine the terms into the forms described in method **(a)** or **(b)**.



Distance between 2 Points

1. Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

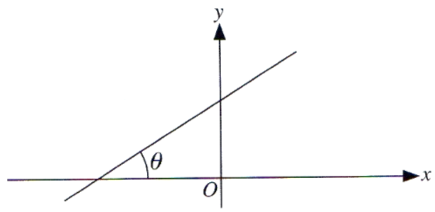
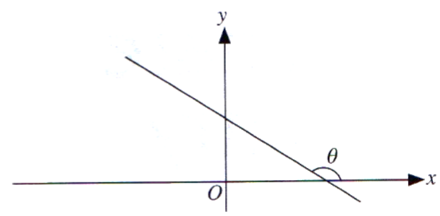
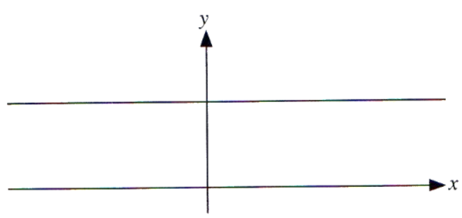
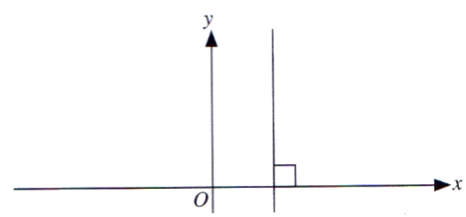
Midpoint of 2 Points

2. Midpoint of $AB = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Gradient of Line and Collinear Points

3. Gradient of AB, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2} = \tan\theta$

4.

$m > 0$ i.e. $0° < \theta < 90°$ (line slopes upwards) 	$m > 0$ i.e. $90° < \theta < 180°$ (line slopes downwards) 
$m = 0$ i.e. $\theta = 0°$ (horizontal line) 	m is undefined i.e. $\theta = 90°$ (vertical line) 

5. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, then gradient of AB = gradient of BC = gradient of AC and area of $\Delta ABC = 0$.

Parallel and Perpendicular Lines

"Finding normal to the tangent" questions

6. If two lines l_1 and l_2 have gradients m_1 and m_2 respectively,
   - l_1 is parallel to l_2 if $m_1 = m_2$.
   - l_1 is perpendicular to l_2 if $m_1 m_2 = -1$.

$m_2 = -1/m_1$

Equation of a Straight Line

7. Gradient form: $y = mx + c$, where m is the gradient and c is the y-intercept

8. Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the intercepts the line makes on the x-axis and y-axis respectively

9. General form: $Ax + By + C = 0$, where A, B and C are constants

Area of Polygons

10. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, …, and $N(x_n, y_n)$ form a polygon, then

$$\text{Area of polygon} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \ldots x_n & x_1 \\ y_1 & y_2 & y_3 \quad\; y_n & y_1 \end{vmatrix}$$

$$= \frac{1}{2}(x_1y_2 + x_2y_3 + \ldots + x_ny_1 - x_2y_1 - x_3y_2 - \ldots - x_1y_n)$$

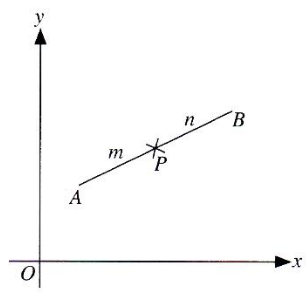
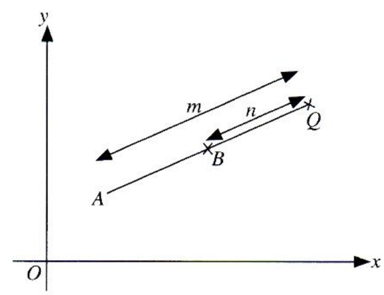
11. If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ form a triangle ABC, then

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

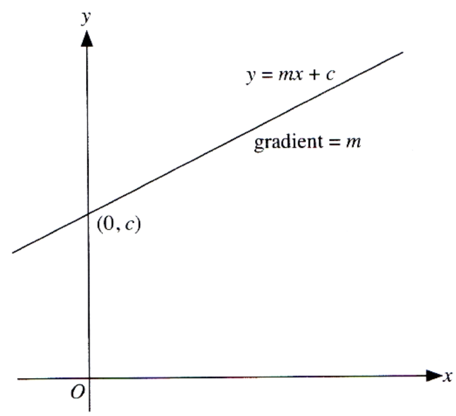
12. Vertices must be taken in a cyclic and anticlockwise order.

Ratio Theorem

13.

Internal point of division	External point of division
Let the point P divide the line AB internally in the ratio $m : n$, then P is the point $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$. 	Let the point Q divide the line AB externally in the ratio $m : n$, then Q is the point $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$. 

1. **The Linear Law**



If the variables x and y are related by the equation $y = mx + c$, then a graph of the values of y plotted against their respective values of x is a straight line graph.

The straight line has a gradient m and it cuts the "vertical" axis at the point $(0, c)$.

2. Linear Law is used to reduce non-linear functions to the linear form $y = mx + c$.

3. To reduce confusion, we sometimes denote the horizontal axis as X and the vertical axis as Y.
i.e. $Y = mX + c$.

4. Some of the common functions and how to linearise them are shown in the table below.

Functions		Plotted along horizontal axis	Plotted along vertical axis
1. $y = ax^n + b$		x^n	y
2. $y = \frac{a}{x^n} + b$		$\frac{1}{x^n}$	y
3. $\frac{1}{y} = ax^n + b$		x^n	$\frac{1}{y}$
4. $y = a\sqrt[n]{x} + b$		$\sqrt[n]{x}$	y
5. $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$	or $y\sqrt{x} = ax + b$	x	$y\sqrt{x}$
6. $xy = \frac{a}{x} + bx$	or $x^2y = bx^2 + a$	x^2	x^2y
7. $x = bxy + ay$	or $\frac{x}{y} = bx + a$	x	$\frac{x}{y}$
8. $\frac{a}{x} + \frac{b}{y} = n$	or $\frac{1}{y} = \left(-\frac{a}{b}\right)\frac{1}{x} + \frac{n}{b}$	$\frac{1}{x}$	$\frac{1}{y}$
or $ay + bx = nxy$	or $y = \frac{a}{n}\left(\frac{y}{x}\right) + \frac{b}{n}$	$\frac{y}{x}$	y
9. $y = ax^2 + bx + n$	or $\frac{y-n}{x} = ax + b$	x	$\frac{y-n}{x}$
10. $y = a^2x^2 + 2abx + b^2$	or $\sqrt{y} = ax + b$	x	$\sqrt{y}$
	or $\sqrt{y} = -ax - b$	x	$\sqrt{y}$
11. $y = \frac{a}{x-b}$	or $\frac{1}{y} = \frac{1}{a}x - \frac{b}{a}$	x	$\frac{1}{y}$
12. $y = ax^b$	or $\lg y = b\lg x + \lg a$	$\lg x$	$\lg y$
	or $\ln y = b\ln x + \ln a$	$\ln x$	$\ln y$
13. $y = n - ax^b$	or $\lg(n-y) = b\lg x + \lg a$	$\lg x$	$\lg(n-y)$
14. $y = ab^x$	or $\lg y = x\lg b + \lg a$	x	$\lg y$
15. $y^n = \frac{b^x}{a}$	or $\lg y = x\left(\frac{\lg b}{n}\right) - \frac{\lg a}{n}$	x	$\lg y$
16. $ya^x = b + n$	or $\lg y = (-\lg a)x + \lg(b+n)$	x	$\lg y$
17. $y^b = 10^{2x+a}$	or $\lg y = \frac{2}{b}x + \frac{a}{b}$	x	$\lg y$
18. $y^b = e^{2x+a}$	or $\ln y = \frac{2}{b}x + \frac{a}{b}$	x	$\ln y$

Equation of a Circle

1. Standard form

$$(x-a)^2 + (y-b)^2 = r^2$$

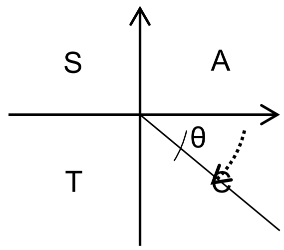
where (a, b) is the centre and r is the radius

2. General form

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

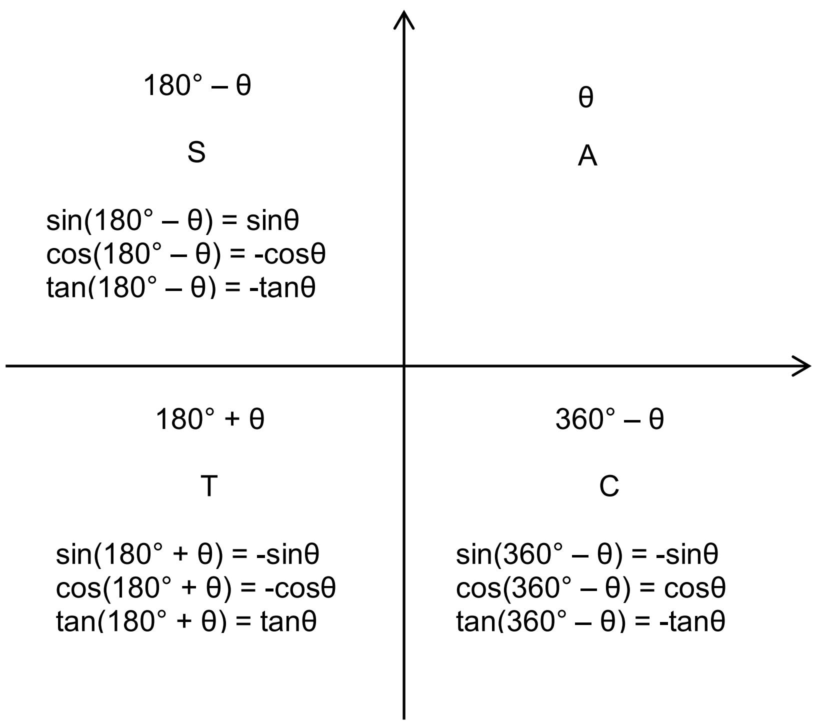
where $(-g, -f)$ is the centre and $\sqrt{g^2 + f^2 - c}$ is the radius

Negative-or-positive Basic Identities



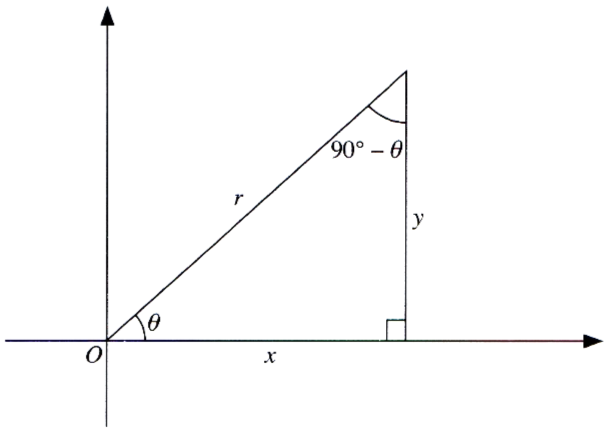
1. cos(-θ) = cosθ
2. sin(-θ) = -sinθ
3. tan(-θ) = -tanθ

ASTC Identities



Basic Trigonometric Ratios

1.



$\sin\theta = \dfrac{y}{h}$	$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{h}{y}$
$\cos\theta = \dfrac{x}{h}$	$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{h}{x}$
$\tan\theta = \dfrac{y}{x}$	$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{x}{y}$

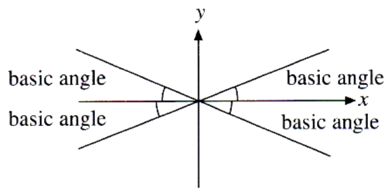
Complementary Angles

2.

$\sin(90° - \theta) = \cos\theta$	$\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$	$\cot(90° - \theta) = \tan\theta$
$\sec(90° - \theta) = \operatorname{cosec}\theta$	$\operatorname{cosec}(90° - \theta) = \sec\theta$

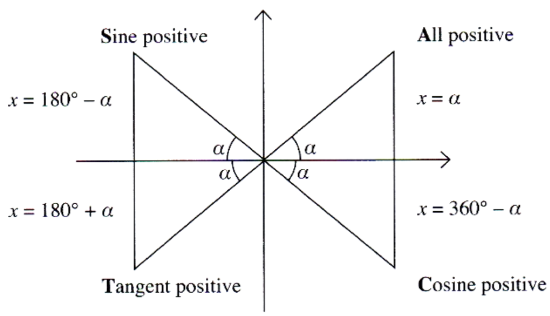
Basic Angle (or Reference Angle)

3. The basic angle, α, is the acute angle between a rotating radius about the origin and the x-axis.



Signs of Trigonometric Ratios in the Four Quadrants

4.



Trigonometric Ratios of Special Angles

5.

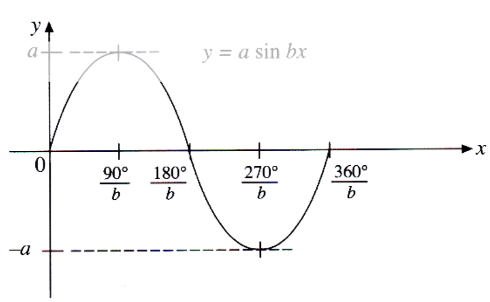
θ	$0°$	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$	$180° = \pi$	$270° = \frac{3\pi}{2}$	$360° = 2\pi$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined	0	Undefined	0

Graphs of Trigonometric Functions

6. $y = a \sin bx$

amplitude $= a$

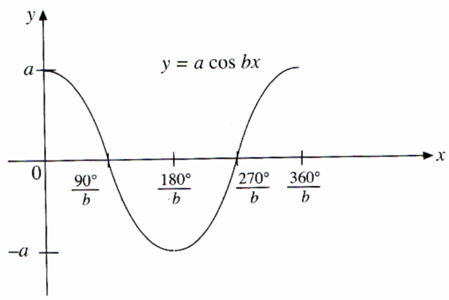
period $= \dfrac{360°}{b}$



7. $y = a \cos bx$

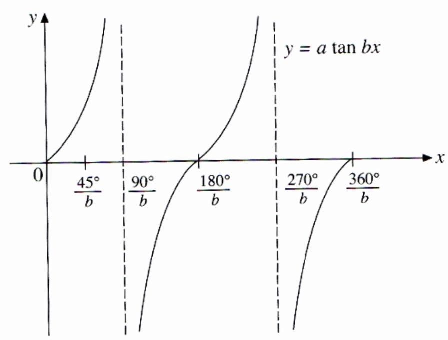
amplitude $= a$

period $= \dfrac{360°}{b}$



8. $y = a \tan bx$

period $= \dfrac{180°}{b}$



9. To sketch the graphs of

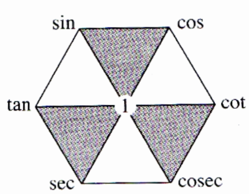
$y = a \sin bx + c$ or $y = a \cos bx + c$ or $y = a \tan bx + c$:

Step 1: Draw the graph of $y = a \sin bx$ or $y = a \cos bx$ or $y = a \tan bx$.

Step 2: If $c > 0$, shift the graph up by c units. If $c < 0$, shift the graph down by $|c|$ units.

Fundamental Identities

10.



11. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

12. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

13. $\sec\theta = \dfrac{1}{\cos\theta}$

14. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

15. $\sin^2\theta + \cos^2\theta = 1$

16. $\tan^2\theta + 1 = \sec^2\theta$

17. $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Curve-sketching General Tips

1. Label: range, amplitude, period
2. Label x-axis, y-axis, origin

Compound Angle Formulae

1. $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$

2. $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

3. $\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Formulae

4. $\sin 2A = 2 \sin A \cos A$

5. $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

6. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Half Angle Formula

Replace A with $\dfrac{A}{2}$ in the Double Angle Formulae.

7. $\sin A = 2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}$

8. $$\begin{aligned} \cos A &= \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} \\ &= 2 \cos^2 \frac{A}{2} - 1 \\ &= 1 - 2 \sin^2 \frac{A}{2} \end{aligned}$$

9. $\tan A = \dfrac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$

Factor Formulae

10. $\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$ $\qquad$ $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

11. $\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$ $\qquad$ OR $\qquad$ $\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

12. $\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$ $\qquad$ $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

13. $\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$ $\qquad$ $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

R-Formulae

$$\left.\begin{array}{ll}
\textbf{14.} & a \sin\theta + b\cos\theta = R\sin(\theta + \alpha) \\
\textbf{15.} & a \sin\theta - b\cos\theta = R\sin(\theta - \alpha) \\
\textbf{16.} & a \cos\theta + b\sin\theta = R\cos(\theta - \alpha) \\
\textbf{17.} & a \cos\theta - b\sin\theta = R\cos(\theta + \alpha)
\end{array}\right\} \text{ where } R = \sqrt{a^2 + b^2} \text{ and } \tan\alpha = \frac{b}{a}$$

18. For the expression $a \sin\theta \pm b\cos\theta$ or $a\cos\theta \pm b\sin\theta$,
    - Maximum value $= R$
    - Minimum value $= -R$,

    when $\tan\alpha = \dfrac{b}{a}$

Formulae

1. $\frac{\mathrm{d}}{\mathrm{d}x}(x^n) = nx^{n-1}$

2. $\frac{\mathrm{d}}{\mathrm{d}x}(ax^n) = anx^{n-1}$

3. $\frac{\mathrm{d}}{\mathrm{d}x}(k) = 0$

Addition/Subtraction Rules

4. If $y = \mathrm{u}(x) \pm \mathrm{v}(x)$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{u}(x)] \pm \frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{v}(x)]$$

Chain Rule

5. If y is a function of u, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

6. $\frac{\mathrm{d}}{\mathrm{d}x}\left[(ax + b)^n\right] = an(ax + b)^{n-1}$

7. In general, $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{f}(x)]^n = n[\mathrm{f}(x)]^{n-1} \cdot \mathrm{f}'(x)$

Product Rule

8. If $y = uv$, where u and v are functions of x, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

Quotient Rule

9. If $y = \frac{u}{v}$, where u and v functions of x, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

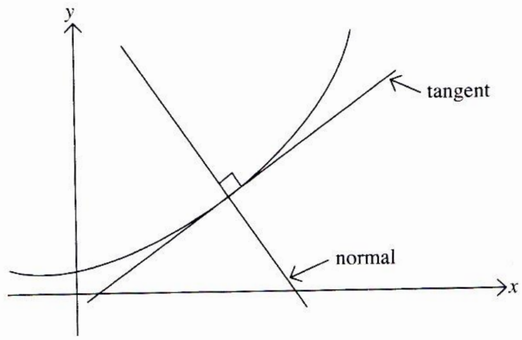
Increasing/ Decreasing Function

10. If y is an increasing function (y increases as x increases), the gradient is positive, i.e. $\frac{dy}{dx} > 0$.

11. If y is a decreasing function (y decreases as x increases), the gradient is negative, i.e. $\frac{dy}{dx} < 0$.

Equation of Tangent and Normal to a Curve

12. Equation of a straight line: $y - y_1 = m(x - x_1)$



13. To find the equation of a tangent, we need:

Gradient of tangent, $m = \frac{dy}{dx}$

Coordinates of a point that lies on the tangent, (x_1, y_1)

14. To find the equation of a normal, we need:

Gradient of tangent $= \frac{dy}{dx}$

Gradient of normal $= -1 \div \frac{dy}{dx}$

Coordinates of a point that lies on the normal, (x_1, y_1)

Higher Derivatives

15.

Function in x	y	$f(x)$
First derivative	$\frac{dy}{dx}$	$f'(x)$
Second derivative	$\frac{d^2y}{dx^2}$	$f''(x)$
Third derivative	$\frac{d^3y}{dx^3}$	$f'''(x)$

Connected Rates of Change

1. If $\frac{dx}{dt}$ is the rate of change of x with respect to time t and $y = f(x)$, then the rate of change of y with respect to t is given by $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$.

2. A positive rate of change is an increase in the magnitude of the quantity involved as the time increases.

3. A negative rate of change is a decrease in the magnitude of the quantity involved as the time increases.

Stationary Points

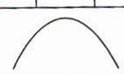
4. If a point (x_0, y_0) is a stationary point of the curve $y = f(x)$, then $\frac{dy}{dx} = 0$ when $x = x_0$, i.e. the gradient of the tangent at $x = x_0$ is zero.

5. A stationary point can be a maximum point, a minimum point or a point of inflexion.

Determining the Nature of the Stationary Points

6. **First Derivative Test**: Use $\frac{dy}{dx}$.

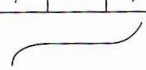
Maximum point

	x^-	x_0	x^+
$\frac{dy}{dx}$	>0	0	<0
slope	/	—	\
stationary point			

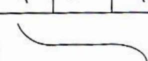
Minimum point

	x^-	x_0	x^+
$\frac{dy}{dx}$	<0	0	>0
slope	\	—	/
stationary point			

Point of inflexion

	x^-	x_0	x^+
$\frac{dy}{dx}$	>0	0	>0
slope	/	—	/
stationary point			

Point of inflexion

	x^-	x_0	x^+
$\frac{dy}{dx}$	<0	0	<0
slope	\	—	\
stationary point			

7. **Second Derivative Test**: Use $\frac{d^2y}{dx^2}$.

If $\frac{d^2y}{dx^2} < 0$, the stationary point is a maximum point.

If $\frac{d^2y}{dx^2} > 0$, the stationary point is a minimum point.

If $\frac{d^2y}{dx^2} = 0$, the stationary point can be a maximum point, a minimum point or a point of inflexion. Use the First Derivative Test to determine the nature.

Problems on Maxima and Minima

8. Step 1: Find a relationship between the quantity to be maximised or minimised and the variable(s) involved.

Step 2: If there is more than one variable involved, use substitution to reduce it to one independent variable only.

Step 3: Find the first derivative of the expression obtained above.

Step 4: Equate the first derivative to zero to obtain the value(s) of the variable.

Step 5: Check the nature of the stationary point.

Step 6: Find the required maximum or minimum value of the quantity.

1. Ensure that your calculator is in radian mode.

Formulae

2.
$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

3.
$$\frac{d}{dx}[\sin(Ax+B)] = A\cos(Ax+B)$$
$$\frac{d}{dx}[\cos(Ax+B)] = -A\sin(Ax+B)$$
$$\frac{d}{dx}[\tan(Ax+B)] = A\sec^2(Ax+B)$$

4.
$$\frac{d}{dx}\left[\sin^n x\right] = n\sin^{n-1}x\cos x$$
$$\frac{d}{dx}\left[\cos^n x\right] = -n\cos^{n-1}x\sin x$$
$$\frac{d}{dx}\left[\tan^n x\right] = n\tan^{n-1}x\sec^2 x$$

5.
$$\frac{d}{dx}[\sin^n(Ax+B)] = An\ \sin^{n-1}(Ax+B)\ \cos\ (Ax+B)$$
$$\frac{d}{dx}[\cos^n(Ax+B)] = -An\ \cos^{n-1}(Ax+B)\ \sin\ (Ax+B)$$
$$\frac{d}{dx}[\tan^n(Ax+B)] = An\ \tan^{n-1}(Ax+B)\ \sec^2\ (Ax+B)$$

In general,

6.
$$\frac{d}{dx}[\sin^n f(x)] = n\ \sin^{n-1}f(x) \times \frac{d}{dx}[\sin\ f(x)]$$
$$\frac{d}{dx}[\cos^n f(x)] = n\ \cos^{n-1}f(x) \times \frac{d}{dx}[\cos\ f(x)]$$
$$\frac{d}{dx}[\tan^n f(x)] = n\ \tan^{n-1}f(x) \times \frac{d}{dx}[\tan\ f(x)]$$

Differentiation of Exponential Functions

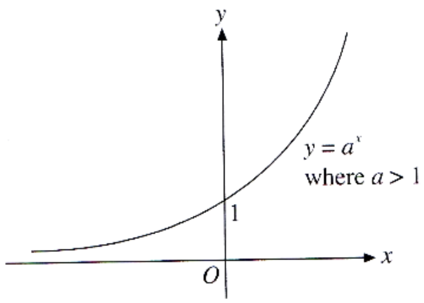
1. $\frac{d}{dx}(e^x) = e^x$

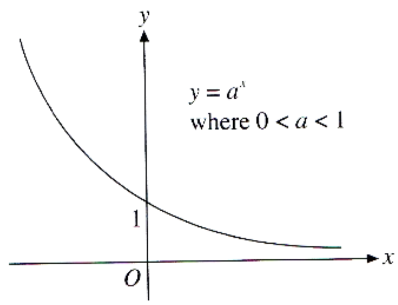
2. $\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$

3. In general, $\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$, where $f'(x) = \frac{d}{dx}f(x)$.

Graphs of Exponential Functions

4.





Differentiation of Logarithmic Functions

5. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

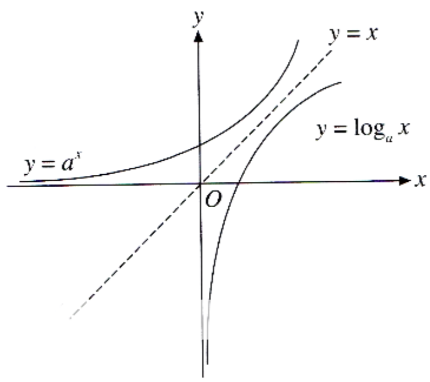
6. $\frac{d}{dx}[\ln(ax + b)] = \frac{a}{ax + b}$

7. In general, $\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$, where $f'(x) = \frac{d}{dx}f(x)$

8. As far as possible, make use of the laws of logarithms to simplify logarithmic expressions before finding derivatives.

Graphs of Logarithmic Functions

9. The graphs of $y = a^x$ and $y = \log_a x$ are reflections of each other in the line $y = x$.



Integration

1. If $y = \mathrm{f}(x)$, then $\int y \, \mathrm{d}x = \int \mathrm{f}(x) \, \mathrm{d}x$.

2. If $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(x)$, then $\int \mathrm{g}(x) \, \mathrm{d}x = y + c$.

Formulae and Rules

3. $\int ax^n \, \mathrm{d}x = \dfrac{ax^{n+1}}{n+1} + c$, where $n \neq -1$

4. $\int (ax + b)^n \, \mathrm{d}x = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$, where $n \neq -1$

5. $\int \mathrm{f}(x) \pm \mathrm{g}(x) \, \mathrm{d}x = \int \mathrm{f}(x) \, \mathrm{d}x \pm \int \mathrm{g}(x) \, \mathrm{d}x$

Integration of Trigonometric Functions

6. $\int \sin x \, \mathrm{d}x = -\cos x + c$

7. $\int \cos x \, \mathrm{d}x = \sin x + c$

8. $\int \sec^2 x \, \mathrm{d}x = \tan x + c$

9. $\int \sin (Ax + B) \, \mathrm{d}x = -\dfrac{1}{A} \cos (Ax + B) + c$

10. $\int \cos (Ax + B) \, \mathrm{d}x = \dfrac{1}{A} \sin (Ax + B) + c$

11. $\int \sec^2 (Ax + B) \, \mathrm{d}x = \dfrac{1}{A} \tan (Ax + B) + c$

12. Methods of Integrating Trigonometric Functions:

- use trigonometric identities e.g. $1 + \tan^2 x = \sec^2 x$
- use double angle formulae e.g. $\cos 2x = 2\cos^2 x - 1$ or $\cos 2x = 1 - 2\sin^2 x$
- use factor formulae

Integration of $\frac{1}{ax+b}$

13. $\int \frac{1}{x}\,dx = \ln|x| + c$

14. $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$

15. In general,

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Integration of e^x

16. $\int e^x\,dx = e^x + c$

17. $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$

Definite Integrals

18. $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$

19. $\int_a^a f(x)\,dx = 0$

20. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

21. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

22. $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

23. $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Integration Using Partial Fractions

Step 1: Check that the fraction is improper

P(x)/Q(x) is a proper algebraic fraction if the degree of P(X) is smaller than the degree of Q(x)

P(x)/Q(x) is a proper algebraic fraction if the degree of P(X) is smaller than the degree of Q(x)

Step 2

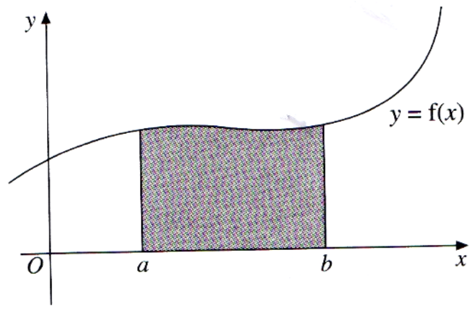
Factorize the denominator

Step 3: Examine the factors of the denominator to decide on the general form of the partial fraction

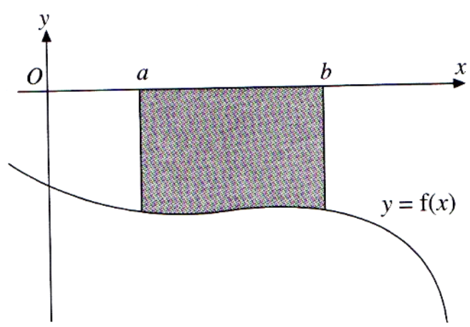
No.	Type of factor	General form	General form of partial fraction
1	Linear factors	ax + b	$\frac{A}{ax+b}+\frac{B}{cx+d}$
2	Irreducible quadratic factors	$ax^2 + bx + c$	$\frac{Ax+B}{ax^2+bx+c}$
3	Repeated linear factors	$(ax + b)^2$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$

Area under a Curve

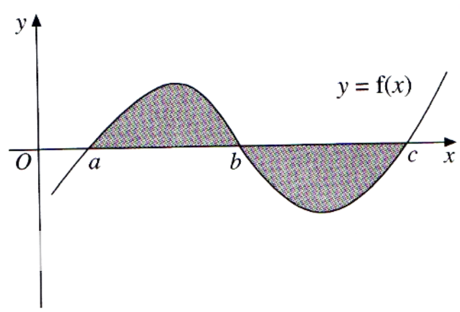
1. **For a region above the x-axis:**
   Area bounded by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is $\int_a^b f(x)\ dx$



2. **For a region below the x-axis:**
   Area bounded by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis is $\left|\int_a^b f(x)\ dx\right|$



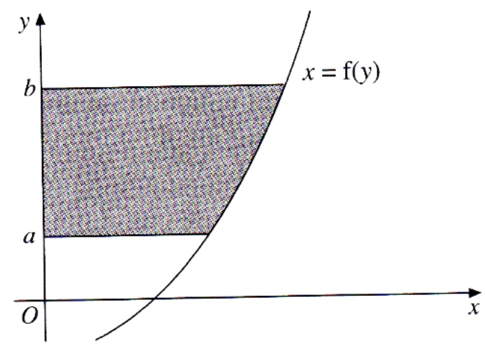
3. **For an area enclosed above and below the x-axis:**
   Area bounded by the curve $y = f(x)$ and the x-axis as shown below is $\int_a^b f(x)\ dx + \left|\int_b^c f(x)\ dx\right|$



Area bounded by the y-axis

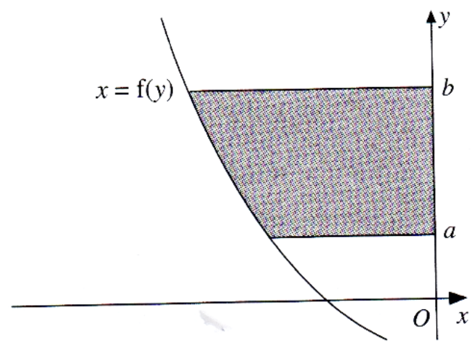
4. For a region on the right side of the y-axis:

Area bounded by the curve $x = f(y)$, the lines $y = a$ and $y = b$ and the y-axis is $\int_a^b f(y)\ dy$



5. For a region on the left side of the y-axis:

Area bounded by the curve $x = f(y)$, the lines $y = a$ and $y = b$ and the y-axis is $\left|\int_a^b f(y)\ dy\right|$



Relationship between Displacement, Velocity and Acceleration

1. Differentiation: $\dfrac{ds}{dt}$ $\qquad \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$

$$\begin{tikzcd} \text{displacement, } s \arrow[r, bend left] & \text{velocity, } v \arrow[l, bend left] \arrow[r, bend left] & \text{acceleration, } a \arrow[l, bend left] \end{tikzcd}$$

Integration: $s = \int v \, dt$ $\qquad v = \int a \, dt$

Common Terms used in Kinematics

2.

Initial	$t = 0$
At rest	$v = 0$
Stationary	$v = 0$
Particle is at the fixed point	$s = 0$
Maximum/minimum displacement	$v = 0$
Maximum/minimum velocity	$a = 0$

3. Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

4. To find the distance travelled in the first n seconds:

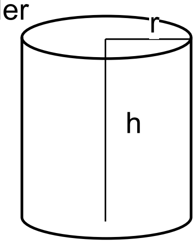
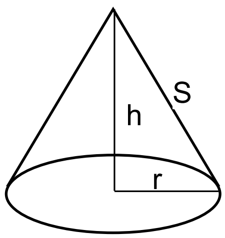
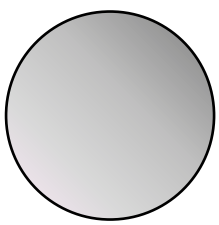
Step 1: Let $v = 0$ to find t.

Step 2: Find s for each of the values of t found in step 1.

Step 3: Find s for $t = 0$ and $t = n$.

Step 4: Draw the path of the particle on a displacement-time graph.

Area and Volume

Shape	Area	Volume
Cylinder 	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$
Cone 	$\pi r^2 + \pi rS$	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Differentiation by first principles

1. **Let (x,y) and (x+δx, y+δy) be 2 points on a curve**
2. **y = f(x), δy = f(δx)**
3. **Equation 2 minus equation 1 to find δy**
4. **Divide δy by δx to find δy/δx**
5. **Set lim to 0**

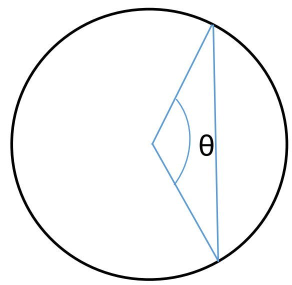
$$\lim_{\delta x \to 0} \left(\frac{\delta y}{\delta x}\right)$$

Miscellaneous

1 radian = $\frac{180}{\pi}$

π radian = 180

2π radian = 360



Area of sector = $\frac{1}{2}\theta r^2$ (θ is in radians)

Arc length, $L = \theta r$ (θ is in radians)

Area of segment = $\frac{1}{2}(\theta - \sin\theta)r^2$ (θ is in radians)