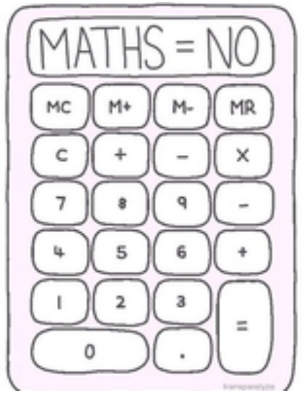
Ew School



ADEY'S HANDY DANDY NOTEBOOK: MATH EDITION

MACROCONCEPTS

Change | Scale | Models | Systems

HELPFUL FORMULAE

STR8 LINE EQUATIONS/POLYNOMIAL EQUATIONS

$y = mx + c$
$y - y_1 = m(x - x_1)$

$(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$
$(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$

THE FORMULA

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HOW TO COMPLETE THE SQUARE

$$x^2 + bx + c = 0$$
$$x^2 + bx = -c$$
$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = d$$
$$x + \frac{b}{2} = \pm\sqrt{d}$$
$$x = \frac{-b}{2} \pm \sqrt{d}$$

HELPFUL FORMULAE

AREA OF TRIANGLE

$\frac{1}{2}r^2\sin\theta$ | $\frac{1}{2}ab\sin\theta$ | $\frac{1}{2}bh$ | $a^2 + b^2 = c^2$

$a/\sin A = b/\sin B = c/\sin C$ [sin rule]

$\cos A = (b^2 + c^2 - a^2)/(2bc)$
$\cos B = (a^2 + c^2 - b^2)/(2ac)$
$\cos C = (a^2 + b^2 - c^2)/(2ab)$

VOL/AREA

Cone Vol: $\frac{1}{3}\pi r^2 h$
Curved Surface of Cone: πr(slant side)
Sphere Vol: $1\frac{1}{3}\pi r^2$
Curved Surface of Sphere: $4\pi r^2$
Cylinder Vol: $\pi r^2 h$
Pyramid Vol: $\frac{1}{3}h$(base area)
Trapezium Area: $\frac{1}{2}h$(sum of parallel sides)

LAWS OF INDICES

$n^0 = 1$

$n^1 = n$

$(n^a) \times (n^b) = n^{a+b}$

$(n^a) / (n^b) = n^{a-b}$

$(n^a)^b = n^{ab}$

$(n^{a/b}) = \sqrt[b]{n^a}$

$(mn)^a = m^a n^a$

$(m/n)^a = m^a / n^a$

INEQUALITIES

$>$ = LARGER THAN
$<$ = SMALLER THAN

$\geq$ = LARGER THAN OR EQUAL TO
$\leq$ = SMALLER THAN OR EQUAL TO

If c is +ve, $a < b$, $ac < bc$
If c is -ve, $a < b$, $ac > bc$

If $a < b$, $-a > -b$ (v.v)

If $a < b$, $1/a > 1/b$ (v.v)

QUADRATIC INEQUALITIES

Option I: Factorise & solve.

Option II: Complete square & solve.

Option III: Use THE FORMULA & solve.

CIRCULAR MEASURE

πrad = 180°

Arc length = $r\theta$

Area of a sector: $\frac{1}{2}(r^2\theta)$

Area of a segment: (area of sector) - (area of triangle)

Equation of a circle: $(a - x)^2 - (b - y)^2 = r^2$ (refer to fig. 1)

If $r = 1$, $\theta = 1$ | a triangle within the semicircle is always a right angle triangle | angle at centre of triangle (refer to fig. 2)

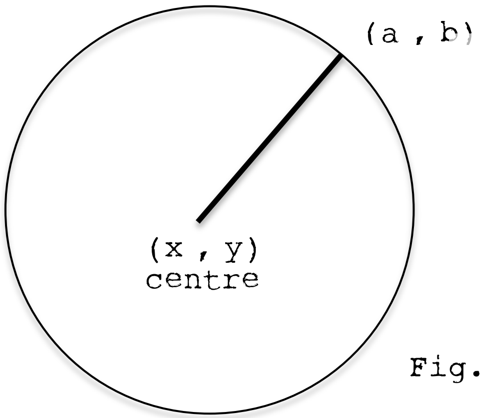


Fig. 1

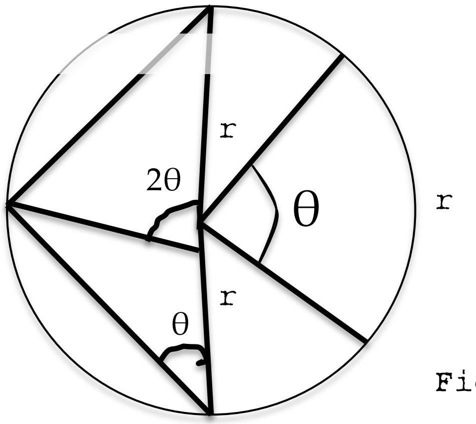
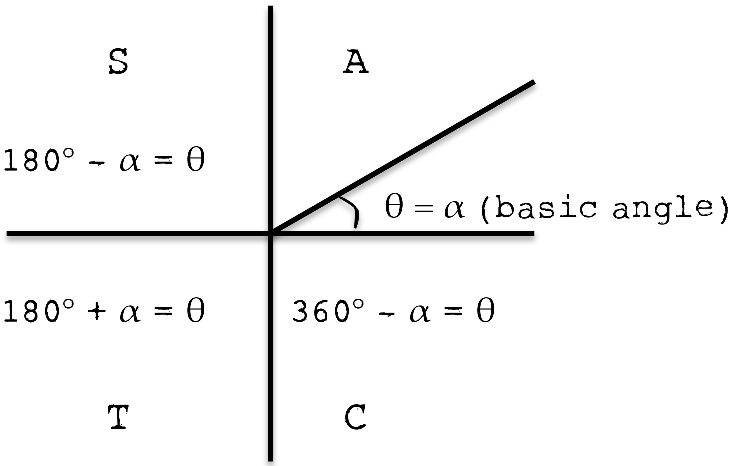
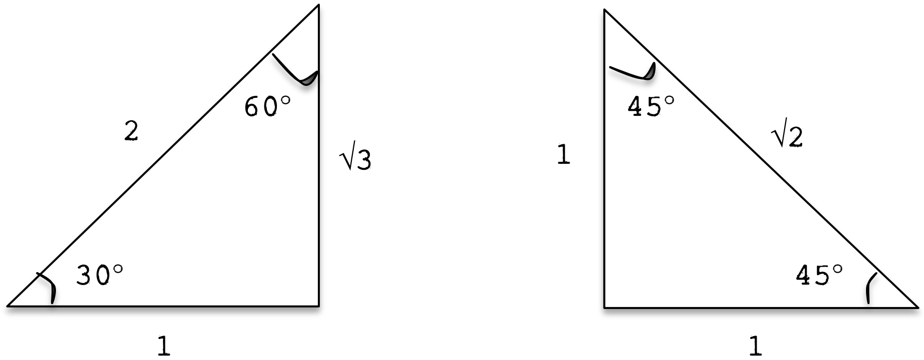


Fig. 2

TRIG III

θ	0°	30°	45°	60°	90°	180°	270°	360°
sinθ	0	½	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	−1	0
cosθ	1	$\sqrt{3}/2$	$\sqrt{2}/2$	½	0	−1	0	1
tanθ	0	$\sqrt{3}/2$	1	$\sqrt{3}$	Undef.	0	Undef.	0





not to scale

TOA
CAH
SOH

$\tan(-\theta) = \cot\theta$
$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$\cos\theta = 1/\sec\theta$
$\sin\theta = 1/\mathrm{cosec}\theta$
$\tan\theta = 1/\cot\theta$

TRIG III

$\tan(90^\circ - \theta) = \cot\theta$
$\cos(90^\circ - \theta) = \sin\theta$
$\sin(90^\circ - \theta) = \cos\theta$
$\tan(180^\circ - \theta) = -\tan\theta$
$\cos(180^\circ - \theta) = -\cos\theta$
$\sin(180^\circ - \theta) = \sin\theta$

$\tan(180^\circ + \theta) = \tan\theta$
$\cos(180^\circ + \theta) = -\cos\theta$
$\sin(180^\circ + \theta) = -\sin\theta$

$\tan(360^\circ - \theta) = -\tan\theta$
$\cos(360^\circ - \theta) = \cos\theta$
$\sin(360^\circ - \theta) = -\sin\theta$

DEGREE TO RADIANS

$180^\circ = \pi$
$x^\circ = (\pi/180^\circ)\cdot(x^\circ)$

TRIG III

THINGS TO TAKE NOTE OF

I. LHS: …
  … : RHS (proven)

II. Put units for EVERYTHING.

III. Always put what you're finding before you write an equation (eg. Area of sector = …)

IV. Always put your sf or dp if necessary

V. On diagrams, write α_a (a.k.a, subscript, not superscript)

HEARTS 4 U xoxo

TRIG III

SUM TO PRODUCT

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A + B) = \frac{tan(a) + tan(b)}{1 - tan(a) \cdot tan(b)}$$

$$\tan(A - B) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$$

HELPFUL COW SAYS: For the bio kids, think of sine as heterozygous & cosine as homozygous!
Tangent always has different operation signs on the numerator & denominator.

TRIG III

PRODUCT TO SUM

$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \sin(A - B)]$

$\sin A\sin B = -\frac{1}{2}[\cos(A + B) - \sin(A - B)]$

?

$\sin X + \sin Y = 2\sin(X+Y/2)\cos(X-Y/2)$

$\sin X - \sin Y = 2\cos(X+Y/2)\sin(X-Y/2)$

$\cos X + \cos Y = 2\cos(X+Y/2)\cos(X-Y/2)$

$\cos X - \cos Y = -2\sin(X+Y/2)\sin(X-Y/2)$

TRIG III

DOUBLE ANGLE FORMULAE

$\sin 2A = 2\sin A\cos A$

$\cos 2A$
$= \cos^2 A - \sin^2 A$
$= 1 - 2\sin^2 A$
$= 2\cos^2 A - 1$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

MULTIPLE ANGLE FORMULAE

Replace 2 with n = all 2s in equation become n/2

FRIENDLY TRASHCAN SAYS: U R NOT TRASH N U R GR9. KEEP GOING!

TRIG III

RANGE

If range is $0° < x < 180°$
Range is $0° < 3x < 540°$

CRUCIAL IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$(\sin\theta)(\cos\theta) = \frac{1}{2}\sin(2x)$

GRAPHING?

If cos4x, it changes the range (in this case, more periods are squished into a 360° cycle).

If 4sinx, it changes the height of the graph (in this case, the graph's max/min point is 4/-4)

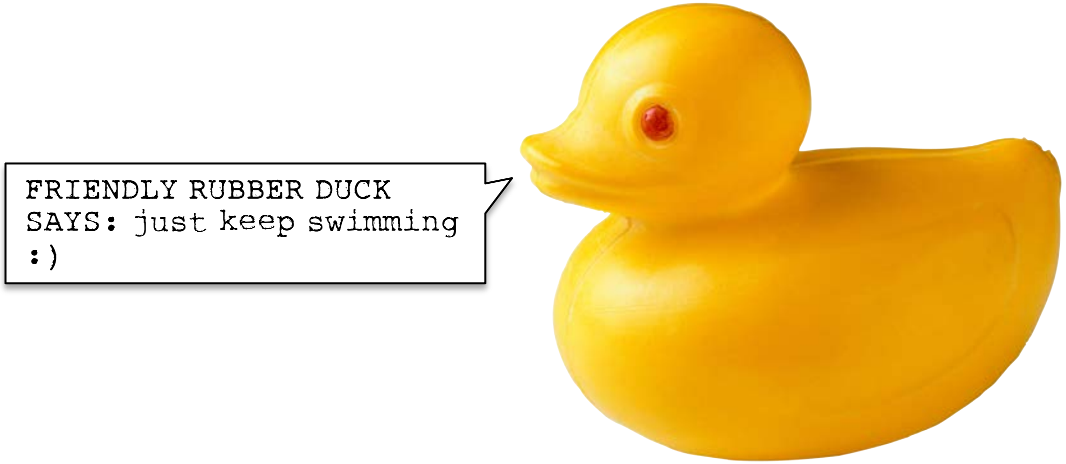
$y = \sin(x \pm \alpha)$: If +, graph moves to LHS. If -, graph moves to RHS.

TRIG III

R-FORMULAE

$$a\sin\theta + b\cos\theta = R\sin(\theta + \alpha)$$
$$a\sin\theta - b\cos\theta = R\sin(\theta - \alpha)$$
$$a\cos\theta + b\sin\theta = R\cos(\theta - \alpha)$$
$$a\cos\theta - b\sin\theta = R\cos(\theta + \alpha)$$

$$\alpha = \tan^{-1}(b/a)$$
$$R = \sqrt{a^2 + b^2}$$



LIMITS

HOW TO FIND LIMITS

Method 1: Substitution (denom. does not become zero if you substitute the number it's tending to)

How: Substitute whatever it's tending to into the given function.

Method 2: Factoring (quadratic)

How: Factorize the quadratic equation and solve.

Method 3: Conjugate (square root)

How: Multiply given function by conjugate to turn the number under a square root into a whole number.

DIFFERENTIATION BY FIRST PRINCIPLES: MS KO'S 5 STEPS TO SUCCESS

Step One:

Take $y = f(x)$ as equation 1'
Take $y + \delta y = f(x + \delta x)$ as equation 2'

Step Two:

2' - 1'. You will obtain δy in terms of x & δx

Step Three:

Divide by δx on both sides. You will obtain $\delta y/\delta x$ in terms of x & δx

Step Four:

Take the limit of $\delta y/\delta x$ as $\delta x \to 0$

Step Five:

You will obtain $\delta y/\delta x = \lim \delta x \to 0(\delta y/\delta x)$ in terms of x for the gradient function of $y = f(x)$

RULES [F(X) | $\delta Y/\delta X$]

Power Rule: x^n | nx^{n-1}

Constant Multiple: ax^n | anx^{n-1}

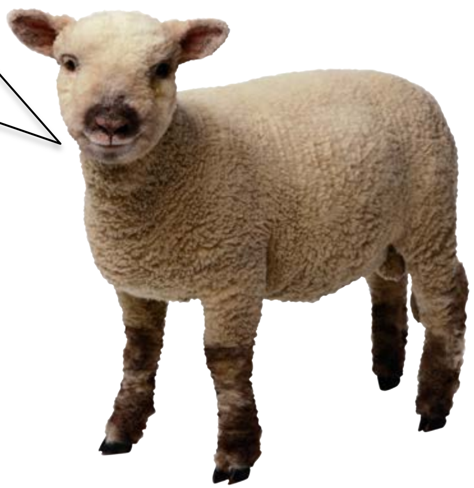
Sum & Difference: $\delta/\delta x[f(x) \pm g(x)] = \delta/\delta x[f(x)] \pm \delta/\delta x[g(x)]$

Chain Rule: $[f(x)]^n$ | $(n[f(x)]^{n-1}) \cdot f'(x)$

Product Rule: $f(x) \cdot g(x)$ | $[f(x) \cdot g'(x)] + [g(x) \cdot f'(x)]$

Quotient Rule: $f(x)/g(x)$ | $\{[g(x) \cdot f'(x)] - [f(x) \cdot g'(x)]\} \div [g(x)]^2$

HELPFUL SHEEP SAYS: an increasing function's gradient function result is > 0 and v.v for a descreasing function.



DERIVATIVES OF TRIG FUNCTIONS

$y = \sin x$, $\delta y/\delta x = \cos x$

$y = \cos x$, $\delta y/\delta x = -\sin x$

$y = \tan x$, $\delta y/\delta x = \sec^2 x$

$y = \sin(ax + b)$, $\delta y/\delta x = a\cos(ax + b)$

$y = \cos(ax + b)$, $\delta y/\delta x = -a\sin(ax + b)$

$y = \tan(ax + b)$, $\delta y/\delta x = a\sec^2 x(ax + b)$

$y = \sin[f(x)]$, $\delta y/\delta x = f'(x)[\cos f(x)]$

$y = \cos[f(x)]$, $\delta y/\delta x = f'(x)[-\sin f(x)]$

$y = \tan[f(x)]$, $\delta y/\delta x = f'(x)[\sec^2 f(x)]$

DERIVATIVES OF TRIG FUNCTIONS

$y = \sin^n x$, $\delta y/\delta x = n(\sin x)^{n-1}(\cos x)$

$y = \cos^n x$, $\delta y/\delta x = -n(\cos x)^{n-1}(\sin x)$

$y = \tan^n x$, $\delta y/\delta x = n(\tan x)^{n-1}(\sec^2 x)$

$y = \sin^n(ax + b)$, $\delta y/\delta x = na[\sin^{n-1}(ax + b)] \text{ x } [\cos(ax + b)]$

$y = \cos^n(ax + b)$, $\delta y/\delta x = -na[\cos^{n-1}(ax + b)] \text{ x } [\sin(ax + b)]$

$y = \tan^n(ax + b)$, $\delta y/\delta x = na[\tan^{n-1}(ax + b)] \text{ x } [\sec^2(ax + b)]$

$y = \sin^n f(x)$, $\delta y/\delta x = n[\sin^{n-1} f(x)][\cos f(x)][f'(x)]$

$y = \cos^n f(x)$, $\delta y/\delta x = -n[\cos^{n-1} f(x)][\sin f(x)][f'(x)]$

$y = \tan^n f(x)$, $\delta y/\delta x = n[\tan^{n-1} f(x)][\sec^2 f(x)][f'(x)]$

LOG DERIVATIVES

First, take note:
$y = \log_a x \Leftrightarrow a^y = x$

$\log_a(mn) = \log_a m + \log_a n$
$\log_a(m/n) = \log_a m - \log_a n$
$\log_a(m^n) = n\log_a m$

$y = \log_a x$, $\delta y/\delta x = 1/x(\log_a e)$

$y = \log_a f(x)$, $\delta y/\delta x = [f'(x)/f(x)](\log_a e)$

$y = \log_e x/\ln x$, $\delta y/\delta x = 1/x$

$y = \ln(ax)$, $\delta y/\delta x = 1/x$

$y = \ln f(x)$, $\delta y/\delta x = f'(x)/f(x)$

$y = \ln[f(x)]^n$, $\delta y/\delta x = n[f(x)'/f(x)]$

EXPO & HIGHER DERIVATIVES

$e^{\ln(x)} = x$
$e^{-\ln(x)} = 1/x$

$y = e^x$, $\delta y/\delta x = e^x$

$y = e^{ax}$, $\delta y/\delta x = ae^{ax}$

$y = e^{ax+b}$, $\delta y/\delta x = ae^{ax+b}$

$y = e^{f(x)}$, $\delta y/\delta x = [e^{f(x)}][f'(x)]$

WHAT ARE HIGHER DERIVATIVES?

Exactly what they sound like. Just keep differentiating, just keep differentiating, just keep differentiating…

EQN. OF TGT AT (X-POINT, INPUT IN PLACE OF X_1) FOR CURVE

Part 1: Differentiate curve.

Part 2: Substitute x-point into $\delta y/\delta x$.

Part 3: This gets you the gradient at the x-point. Input in place of m.

Part 4: Substitute x-point into the curve.

Part 5: This get you the y-coordinate. Input in place of y_1.

Part 6: Do $y - y_1 = m(x - x_1)$

NOTATION

$\delta y/\delta x|x = 2$ means gradient at point $x = 2$

STATIONARY POINTS

These occur when $\delta y/\delta x = 0$; a.k.a, gradient = 0

THREE BASIC TYPES:

1. Minimum/Maximum (quadratic)

2. Inflexion (cubic & above)

3. Local maximum (cubic & above)

<u>HOW TO FIND A STAT PT</u>

Step One: Derive the gradient function.

Step Two: Find the values wherein it equates zero.

Step Three: Solve!

TESTING POINTS

Method One: Second Derivative

Step One: Derive second derivative.

Step Two: Determine whether graph is min/max/inflexion.

If second derivative is POSITIVE, it is a minimum graph.
If second derivative is NEGATIVE, it is a maximum graph.
If second derivative is ZERO, check again with Method Two.

Method Two: Table Method

Template table

x	n^- (slightly smaller value)	n (value)	n^+(slightly larger value)
Sign of $\delta y/\delta x$	+ve/-ve	0	+ve/-ve
Sketch of tgt graph	Upwards if postive	Flat when zero	Downwards if negative

If +ve/0/+ve or -ve/0/-ve, it is a point of inflexion.
If +ve/0/-ve, it is a maximum point.
If -ve/0/+ve, it is a minimum point.

SOLVING CURVE SKETCHING

Step One: Differentiate the given function.

Step Two: At stationary point, $\delta y/\delta x = 0$. Make $\delta y/\delta x = 0$.

THIS GIVES YOU: x-coordinate of turning point.

Step Three: Take x-value and substitute it into original function.

THIS GIVES YOU: coordinates of turning point.

Step Four: Second derivative/table method. Substitute table method figures into differentiated function; substitute second derivative figures into second-derivative function.

THIS GIVES YOU: nature of turning point.

Step Five: When $x = 0$; when $y = 0$. Substitute into original function.

THIS GIVES YOU: where the x/y-axis cut.

STAIRS FOR CURVE SKETCHING

Shape: characteristic shape of graph for particular family of curves

Turning point: nature of stat/minmax/inflexion points

Asymptote: position of asymptote

Intercepts: coordinates of any axial intercepts

Region: domain, range & terminal points

Symmetry: presence of symmetry

RATE OF CHANGE

INTERPRET

$\delta y / \delta x = 5$

This means that y is increasing at the rate of 5 units for every increase of 1 unit in x. A negative = decreasing.

HOW TO SOLVE

Step 1: Define variables.
Step 2: State givens in symbolic form.
Step 3: Identify what is to be solved in symbolic form.
Step 4: Determine the relationship between the given variables.
Step 5: Use suitable substitution to ensure a relationship between TWO variables only.
Step 6: Differentiate whatever you get from step 5.
Step 7: Apply chain rule.
Step 8: Substitute and find unknown.
Step 9: Write statement.

REMEMBER

Always write units!
Define fully.

PARTIAL FRACTIONS

If numerator degree > denominator degree, do long division first, leave the answers as CONSTANT + remainder/divisor and do P.F. as usual.

SUBST

$$\frac{p(x)+q}{(ax+b)(cx+d)} \equiv \frac{A(cx+d)+B(ax+b)}{(ax+b)(cx+d)}$$

$$\therefore p(x)+q \equiv A(cx+d)+B(ax+b)$$

SQUARED TERM

$$\frac{9x^2+2x+14}{(ax+b)(cx-d)^2} \equiv \frac{A}{(ax+b)} + \frac{B}{(cx-d)} + \frac{C}{(cx-d)^2}$$

$$\equiv A(cx-d)^2 + B(ax+b)(cx-d) + C(ax+b)$$

IRREDUCIBLE FACTORS

$$\frac{p(x)+q}{(ax+b)(cx^2+dx+e)} \equiv \frac{A}{(ax+b)} + \frac{Bx+C}{(cx^2+dx+e)}$$

INTEGRATION

ANTI-DERIVATIVE

$\int f(x)dx = F(x) + C$, where C is an arbitrary constant

dx means integrated with respect to x
$F(x) + C$ is the indefinite integral of $f(x)$

INTEGRATION RULES

$dy/dx = ax^n$, $y = (ax^{n+1}/n+1) + C$

$\int kf(x)dx = kF(x) + C$
$\int f(x) + g(x)dx = F(x) + G(x) + C$
$\int f(x) - g(x)dx = F(x) - G(x) + C$

INTEGRATION OF SPECIFIC THINGS

$\int \cos x = \sin x + C$
$\int \sin x = -\cos x + C$
$\int \tan x = -\ln|\cos x| + C$
$\int \sec^2 x = \tan x + C$

$\int \cos(ax + b) = (1/a)\sin(ax + b) + C$
$\int \sin(ax + b) = -(1/a)\cos(ax + b) + C$
$\int \tan(ax + b) = -(1/a)\ln|\cos(ax + b)| + C$
$\int \sec^2 x(ax + b) = (1/a)\tan(ax + b) + C$

$\int \sin^2 x = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$ $[(1 - \cos 2x)/(2)dx = \sin^2 x]$
$\int \cos^2 x = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$ $[(1 + \cos 2x)/(2)dx = \cos^2 x]$
$\int \tan^2 x = \tan x - x + C$ $[\tan^2 x + 1 = \sec^2 x]$

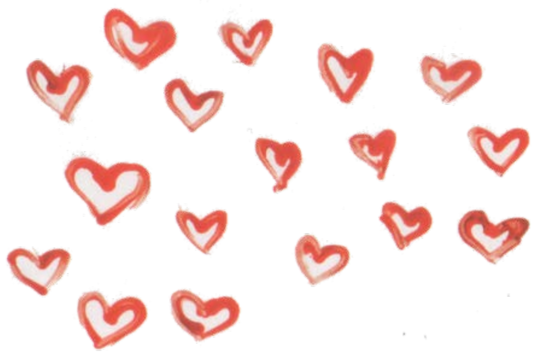
$\int 1/x = \ln|x| + C$
$\int 1/ax + b = (1/a)(\ln|ax + b|) + C$

$\int e^x = e^x + C$
$\int e^{-x} = -e^{-x} + C$
$\int e^{ax + b} = (1/a)e^{ax + b} + C$

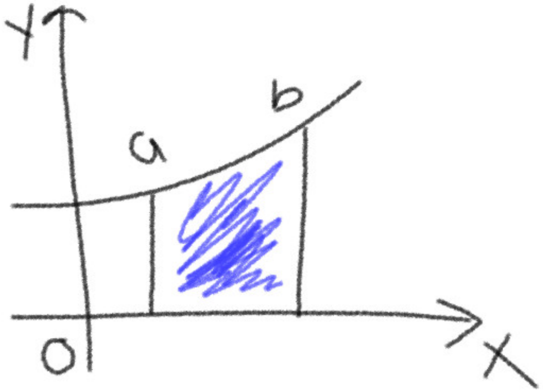
DEFINITE INTEGRALS

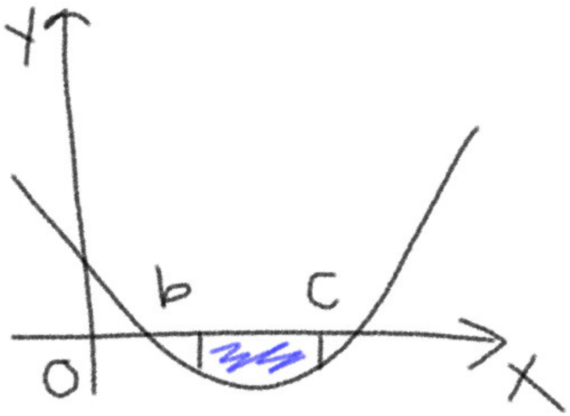
$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

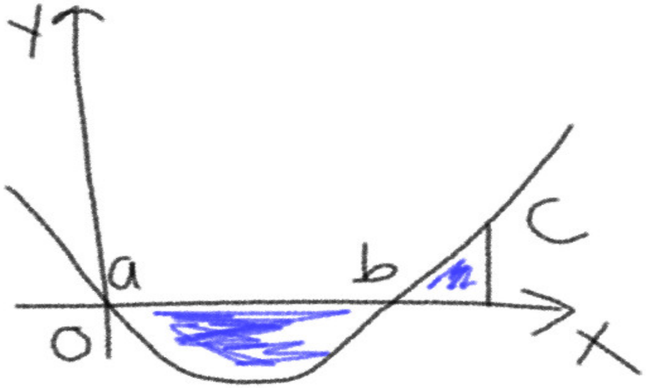
b = upper limit
a = lower limit
f(x) = integrand



AREA BETWEEN CURVE & AXIS





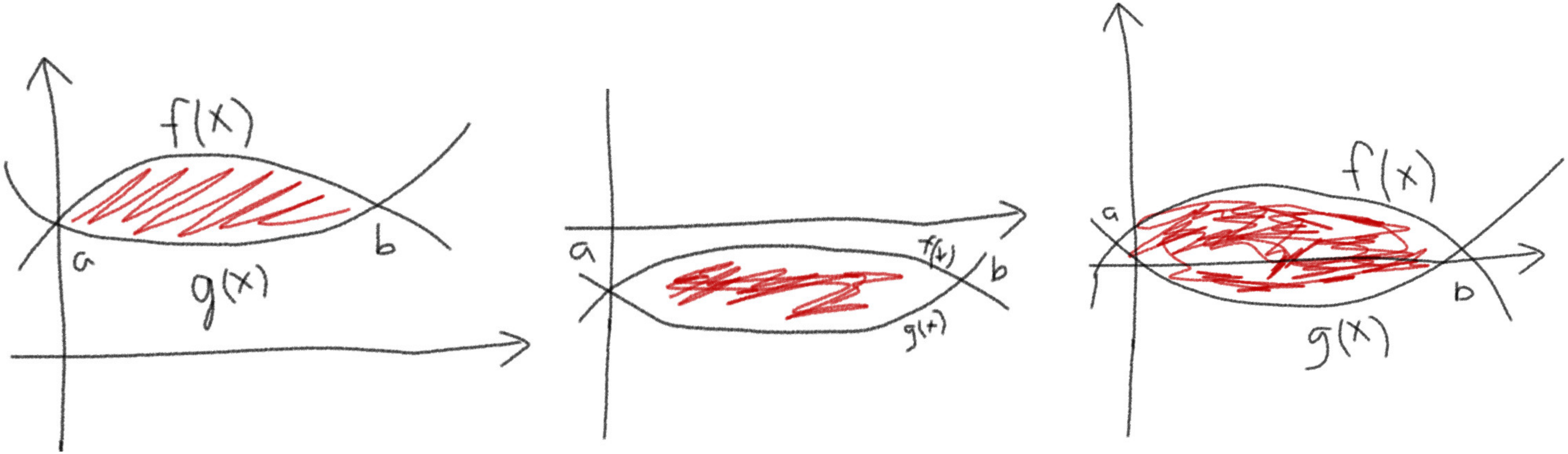


$$\int_a^b y\,dx$$

$$\left|\int_b^c y\,dx\right|$$

$$\int_a^b y\,dx + \left|\int_b^c y\,dx\right|$$

AREA BETWEEN CURVES



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

(I forgot to label my x/y axes and origin. Don't do that in TAs yah)

GRAPHS AND STUFF

TRANSLATION

$y = f(x) + k, k > 0$ $(a,b) \rightarrow (a,b+k)$
graph of f(x) is translated vertically upwards by k units to obtain graph of f(x) + k

$y = f(x) - k, k > 0$ $(a,b) \rightarrow (a,b-k)$
graph of f(x) is translated vertically downwards by k units to obtain graph of f(x) - k

REFLECTION

$y = -f(x)$ $(a,b) \rightarrow (a,-b)$
graph of f(x) is reflected about the x-axis to obtain graph of -f(x)

STRETCH

$y = kf(x), k > 1$ $(a,b) \rightarrow (a,kb)$
graph of kf(x) is stretched parallel to y-axis by stretch factor k to obtain graph of kf(x)

$y = kf(x), 0 < k < 1$ $(a,b) \rightarrow (a,kb)$
graph of kf(x) is compressed parallel to y-axis by stretch factor k to obtain graph of kf(x)

♡ WELL DONE! YOU MADE IT! ♡