Common Mistakes by Topic

DIFFERENTIAL EQNS

- make sure $\frac{dy}{dx}$ is negative if decreasing
- remember to add c/d after integrating!
- remember to sub $\pm e^c$ with A / another constant ⇒ then just find value of A
- if $\frac{dy}{dx} = y(1-y)$, divide $y(1-y)$ on both sides
- to sketch family of curves, it is usually -1, 0, 1 for constants
- no. of stationary points depend on no. of solution to $\frac{dy}{dx}$
- note which are constants and which are variables
- when sketching graphs, note the domain given by the question + the context (eg pop cannot be less than 0)

BINOMIAL EXPANSION

- make sure the expansion term is grouped as constant vs variable eg. $(1+(x^2+x^3))^{-1}$
- make sure to factorise out such that the constant term = 1 (not x).
- in considering validity of expansion, use the whole x-term after factorisation eg. if $(3+x^2)^{-1} \rightarrow \frac{1}{3}(1+\frac{x^2}{3})^{-1}$, then consider $|\frac{x^2}{3}|<1$
- if $(1-x)^{-1}$ [both -ve], then coeff of all terms positive
  if $(1+x)^{-1}$, then coeff of terms is +ve, -ve, +ve...
- if you see partial fractions, use series expansion
- For $(1+x)^n$, the $(r+1)$th term has coeff $\frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$.
- For expansions involving $(x-1)(x+1)^{-1}$, remember that $(x-1)$ will not be part of the coeff
- For substitutions to find values, choose value with a smaller magnitude and those that are within valid range
- For $(1+x)^n$, if $n>0$, $(1+x)^n = a+b+\dots+c$ ⇐ must have ending (series does not go on infinitely)
- if expansion series has a constant eg. $(1+x)^n$, x of all sorts of powers will be produced; however if it is purely x terms eg. $(x^2+x)^n$, then lowest power in the series will be the lowest power in the expression to the power of n ie. x^n.

MACLAURIN SERIES

- remember to divide each differential term by the corresponding factorial

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- also factorise out the constant to make sure the constant = 1 eg. $\ln(2+x) = \ln 2 + \ln(1+\frac{x}{2})$
- for linear approximation, just take the terms which will form a line ☺
- $\sin x \approx x$ $\cos x \approx 1-\frac{x^2}{2}$ $\tan x \approx x$
- if it is $\sin kx$, can just use formula; if it is $\sin(x+\theta)$, then we compound ∠ formula
- remember that denominator can be expressed as power -1
- power of the series must be positive real numbers (not fractions)
- For double approximation, it is usually inside to outside
- if you see a $\sin x$, $\cos x$, e^x, $\ln(1+x)$ with a variable (x), then consider maclaurin series

INEQUALITIES & SLES

- When a number is multiplied by a negative number, inequality sign switches; numbers remain in their positions; reciprocal also flip sign
- draw number lines and use shaded or empty circles
- consider intersection of number lines if it is AND; if it is OR, don't need eg. $x<b$ or $x>b$ means all $\mathbb{R}$
- draw graphs to find inequality if function cannot be easily drawn / of higher power
- in using GC to plot graphs, try to make one side = 0. Then key in y_2 f(x) and y=0 before using intersect command to find answer
- for SLEs, remember $\frac{dy}{dx}=0$ can also form another equation

FUNCTIONS

- be careful when to use square brackets or round brackets
- use shaded or empty circles for graphs
- can use GC to set domain for x !!
- to determine if it is a function → use vertical line test; to determine if it is 1 to 1 → use horizontal line test
- use domain to determine if it is $\pm\sqrt{x+a}$
- principal domain of $\sin x$ and $\tan x$ is $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$; for $\cos x$ it is $0 \le x \le \pi$
  * $\sin^{-1}(x) \neq \frac{1}{\sin x}$; same for cos and tan

- n drawing f^{-1} graph, make sure f and f^{-1} intersect at $y=x$.
- $f^{-1}f = x$ and $ff^{-1} = x$ unless domain & range don't match

- to find solution of $f(x) = f^{-1}(x)$, use $f(x) = x$ instead
- for qns that involve $f^6(x) = f^{-1}(x)$, then find what is $f^{2017}(x)$, always find what value of n in f^n gives x first. Then divide the desired number by n. E.g. in this case, $f^7(x) = x$. $\therefore 2017 \div 7 = 288$ r 1 $\therefore f^{2016} = x$; $f^{2017} = f(x)$.
- to check for equivalent functions must check domain as well

DIFFERENTIATION

- rmbr that $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$; $\frac{d}{dx}\sec x = \sec x \tan x$; $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- for $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, remember to differentiate the x as well!!
- rmbr that $\frac{d}{dx}\log_a f(x) = \frac{f'(x)}{f(x)\ln a}$; $\frac{d}{dx} a^{f(x)} = (f'(x))(a^{f(x)})(\ln a)$
- if you want to differentiate $\left(\frac{dy}{dx}\right)^2$, treat it as a variable; $\therefore \frac{d}{dx}\left(\frac{dy}{dx}\right)^2 = 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$
- for parametric equations, use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ or plot a graph and use GC to calculate
- rmbr gradient of a line can also be represented as $\tan\theta$
- for rate of change, remember to put -ve sign if it is decreasing
- for maxima, minima, use first derivative or 2nd derivative to check
- discern which is constant, which is variable
- differentiate CAREFULLY ⊙.⊙
- find two equations to link 2 variables together
- if $f''(x) > 0$, graph is ∪ ; if $f''(x) < 0$, graph is ∩
- for graph of f(x) vs f'(x), straight line in f'(x) only if f(x) is a quadratic curve; vertical asymptote remains; horizontal /diagonal asymptote depends on differentiation.

SEQUENCES & SERIES

- read context carefully for SNS questions — try to deduce a trend
- for recurrence relation type of question, for algebraic method, try to form an expression with factors eg. quadratic
- to prove formula of summation of GP, consider $S_n - rS_n$
- no. of terms in sigma notation is $(n-m+1)$
- For sigma notation, beware of which terms actually vary (eg. $\sum_{r=1}^{n} r$) and which don't (eg. $\sum_{r=1}^{n} a$)
- if they tell you abt partial fractions, likely MOD

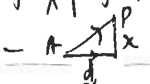
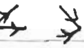
- For qns that ask you to use a summation and prove another inequality, try to draw parallels and connect both summations together eg. for summation, usually it is to change start /end terms ; if fraction is totally different, try to draw links btwn denominators /replace things (idk no tips for this >.<)
- for MI, don't lose marks!!. First statement should include for $n \in \mathbb{Z}^+$, $n \geq 1$ etc
- sometimes the base case may not be 1.

VECTORS

- rmbr that $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$; $\underset{\sim}{a} \times \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \sin\theta \, \hat{\underset{\sim}{n}}$

  $\underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$ (linear algebra!!)

  $\perp$ vectors: $\underset{\sim}{a} \times \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|$; // vectors : $\underset{\sim}{a} \times \underset{\sim}{b} = 0$

  length of projection vector : $|\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}|$
-  $x = |AP \times \hat{\underset{\sim}{d}}|$
- reflection of a pt in a plane → use mid-pt flerem
- for case of no solution, all 3 planes parallel, 2 planes intersecting or 
- if it is , note that the third plane is parallel to the line of intersection btwn other 2 planes
- cross product and dot product also can factorise
- work out what you know & what you don't know keep calm!!.
- when using the vectors for dot product, both vectors must be pointing in or out 
- for eqn of line / plane with parameter, remember to write $\lambda \in \mathbb{R}$, $\mu \in \mathbb{R}$ etc
- read the parameters given in the qn and don't repeat them
- vector eqn of x-axis : $\lambda_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; y-axis : $\lambda_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$; z-axis : $\lambda_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$; $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$
- if you encounter a variable such as st, just group it as w and solve normally → no sweat!!
- OYZ / y-z plane contains OY and OZ ⇒ $\pi : \underset{\sim}{r} = \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$; $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$

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- when there is projection vector, think of this diagram 
- in drawing diagrams, assume the angle is not 90° first

GRAPHING TECHNIQUES

- vertical asymptote → consider denominator ; horizontal asymptote → consider partial fraction ; oblique asymptote → consider partial fractions ; present when numerator is 1 power higher than denominator
- $y=e^x$ → horizontal asymptote ; $y=\ln x$ → vertical asymptote
- $\frac{x+a}{x+b}$ → horizontal & vertical asymptote ; $y=\frac{x^2+a}{x+b}$ → oblique & vertical
- for circles, RHS $=r^2$ (does not need to be equal to 1)
- for ellipse, same family as circles (x^2+y^2) ; RHS $=1$; $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$; b = radius parallel to y-axis ; a = radius parallel to x-axis
- be careful when completing the square
- for hyperbola, ① $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or ② $\frac{(y-h)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$; RHS always $=1$ [① – ② –]
  oblique asymptote : $\frac{(y-k)^2}{b^2} \to \frac{(x-h)^2}{a^2}$; (h,k) is intersection of asymptotes
- parabola : $(y-k)^2=p(x-h)$ $p<0$ ⊃ or $p>0$ ⊂ ; $(x-h)^2=p(y-k)$ ∪ ∩ * (h,k) is the turning pt of the parabola
- for ellipse & hyperbola, denominator under x^2/y^2 corresponds to length parallel to x-axis and y-axis
- parametric, can use GC to set limits
- rmbr to mark out coord of intercepts, stationary pts etc
- use words such as scale / translate / reflect in the positive / -ve x/y direction
- if you stretch than translate, the whole x and the scale factor will be affected by translation $x \to \frac{x}{2} \to \frac{x+2}{2}$
  vs translate than stretch ($x \to x+2 \to \frac{x}{2}+2$) [same for y, but must work out on y-side bef transposing to x-side) → y=0
- for square root function, vertical asymptote remains ; horizontal will become √ ; oblique can just ignore
- for x-intercept, non-repeated root tangent change from oblique to vertical ; repeated root changes from horizontal to oblique (note : only if it is x-intercept!!!)
- for reciprocal, vertical asymptote → x-intercept (& vice versa) ; horizontal will become $\frac{1}{\ }$; oblique → y=0
  min pt → max pt (& vice versa) ; if $f(x)$ is ↑, then $\frac{1}{f(x)}$ ↓ and vice versa
- if translation / scaling is done on eqn of circle / ellipse etc, note the power 2.
- rmbr that area under graph changes with transformation
- beware of 2 types of qns — ① ask you to find values of y where there are no points on graph (use GC to find turning pts or equate the graph to a constant eg. $\frac{x^2+3x}{x-1}=k$ → use discriminant)
  ② find range of values of k with 2 stationary pts ($y=\frac{x^2+kx+1}{x-2}$) → use $\frac{dy}{dx}$

COMPLEX NUMBERS

- can compare Re and Im parts only if unknowns are real
- remember to only include coefficients when finding modulus / argument
- $(z_1 \pm z_2)^* = z_1^* \pm z_2^*$
- $zz^* = |z|^2$
- r in polar & exponential form must always be >0
- when sketching roots in an argand diagram, remember to label the k, circles, dotted lines, modulus & angles
- can consider complex numbers as vectors sometimes
- for argand diagram, it is the coordinates that are labelled (X the complex no.)
- keep calm and see which type of loci it is
- for circle: label center & radius ; for ⊥ bisector: label right angle; start pt & end pt; dotted line for half line: empty circle; angle
- shade the correct value for z esp if it occupies 2 loci
- recognise polar exponential form $e^{i\theta}$ and $e^{-i\theta}$ ⇒ summation = 2Re
- can consider factorising $e^{i\frac{\theta}{2}}$ to form conjugate pairs
- for loci, sometimes may have to factorise out imaginary number to make coeff of z = 1.
- can use tan of arg to find gradient of half line.

INTEGRATION

- rmbr to put mod sign for ln where appropriate
- always try to convert a fraction into a summation of a few fractions
- For $\int \frac{1}{x^2 \pm a^2}$ and $\int \frac{1}{a^2 - x^2}$ (and $\int \frac{1}{\sqrt{a^2-x^2}}$), if coeff of x is not 1, must remember to divide answer by coeff.
- remember to add +c !!
- for substitution, remember to sub back
- for integration by parts, nature of u and v is usually different
- if it is $\int |f(x)|\, dx$, then must split $\int f(x)\, dx + \int -f(x)\, dx$
- for area under graph (in terms of the rectangles), beware which point on the curve refers to which height of the rectangle
- if it is to find area for parametric equations, then must sub in parameter

- $\sqrt{\sin^2\theta} = |\sin\theta|$ (evaluate +ve/-ve based on range)

OTHERS

- when they ask you for locus of mid-point, equate/sub the x and y together by parameters to get the ans

- $\sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- $\text{Area} = \frac{1}{2}ab\sin C$
- $\cos C = \frac{a^2+b^2-c^2}{2ab}$
- ${}^nP_r = \frac{n!}{(n-r)!}$; ${}^nC_r = \frac{n!}{r!(n-r)!}$