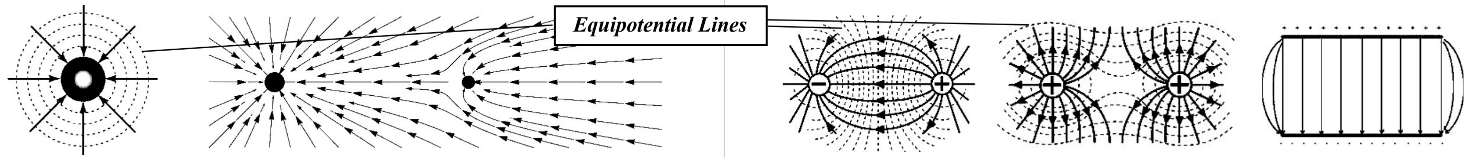
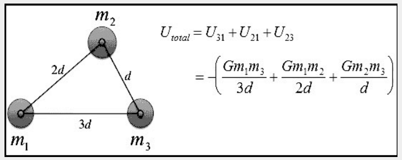
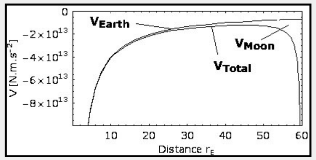
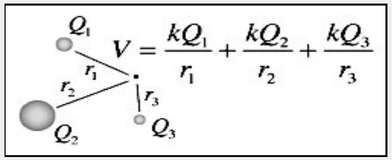
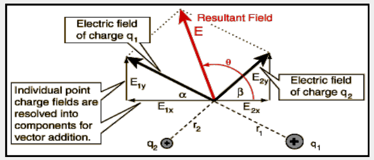
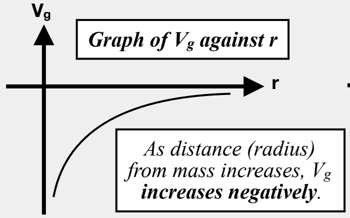
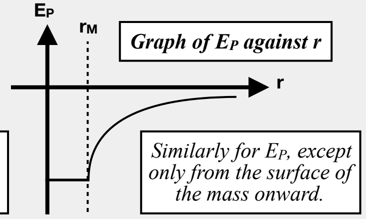
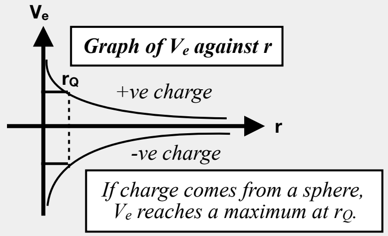
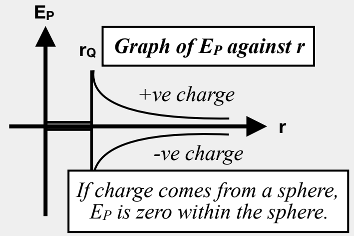
10.1-10.2 Describing Fields, Fields at Work

(A) Comparing Fields

Gravitational and Electric Fields behave similarly. These are the differences in their characteristics:

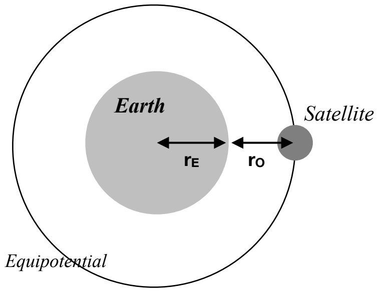
	Gravitational Field	Electric Field
Type of Field	 *G-Field by 1 Mass* *Interaction of G-Fields*	*Opposite Charge E-Field* *Like Charge E-Field* *Parallel Plate E-Field*
*Force, **F***	**Gravitational Force** - An **attractive** force exerted by an object on other objects in its field by virtue of its mass. $\boxed{F_g = \frac{GMm}{r^2}}$ *where **G** is the Gravitational Constant; **M** is the mass of the primary field-producing object and **m** is the test mass.* ****follows Newton's Law of Gravitation** (Topic 6.2)*	**Electrostatic Force** - An **attractive or repulsive** force exerted by an object on other objects in its field by virtue of its mass. $\boxed{F_e = \frac{kQq}{r^2}}$ *where **k** is the Coulomb Constant; **Q** is magnitude of the primary field-producing charge and **q** is the magnitude of the test charge.* ****follows Coulomb's Law of Electrostatics** (Topic 5.1)*
*Field Strength, **g** or **E***	**Gravitational Field Strength** - Force Exerted by Field Per Unit Mass $\boxed{g = \frac{F_g}{m} = \frac{GM}{r^2}}$	**Electric Field Strength** - Force Exerted by Field Per Unit Charge $\boxed{E = \frac{F_e}{q} = \frac{kQ}{r^2}}$
*Potential, **V***	**Gravitational Potential** - Potential Energy per unit test mass exerted by the field. $\boxed{V_g = -\frac{GM}{r}}$ **Gravitational Potential Difference** - Work done per unit mass in moving an object between two points. $\boxed{\Delta V_g = \frac{W}{m} = -\frac{GM}{\Delta r}}$ **Gravitational Potential Gradient** measures the change in Potential, V with respect to Radius, r and is equal to ***-g***. $\boxed{\frac{\Delta V}{\Delta r} = -g}$ ****Work Done, mΔV, is negative as Potential Energy increases when distance falls.***	**Electric Potential** - Potential Energy per unit test charge exerted by the field. $\boxed{V_e = \frac{kQ}{r}}$ **Electric Potential Difference** - Work done per unit charge moving an object between two points. $\boxed{\Delta V_e = \frac{W}{q} = \frac{kQ}{\Delta r}}$ **Electric Potential Gradient** measures the change in Potential, V with respect to Radius, r and is equal to ***-g***. $\boxed{\frac{\Delta V}{\Delta r} = -E = \frac{W}{qr}}$ ****Sign of Work Done, qΔV, follows the Sign of the Charge.***
*Potential Energy, **E**P.*	**Gravitational Potential Energy** is the work done by an external force in moving a test mass from infinity to a point in the field. $\boxed{E_P = mV_g = -\frac{GMm}{r}}$	**Electric Potential Energy** is the work done by an external force in moving a test charge from infinity to a point in the field. $\boxed{E_P = qV_e = \frac{kQq}{r}}$
Interaction of Multiple Fields	 	 

Type of Field	Gravitational Field	Electric Field
Graphical Representation of Potential & Potential Energy	 	 

Important Notes:

1. Work is done when an object moved along the field line. No work is done if movement is perpendicular to the field-induced force.
2. The common approximation g = 9.8ms^{-1} is only able to be used at the Earth's surface and not in space.
3. The other main field which does work is Magnetic Field, but does not influence objects the same way as Gravitational and Electric Field.

(B) G-Field: Satellite Orbital Motion



Geostationary Orbit

A special case of Geosynchronous orbit where the satellite does not appear to move if viewed from the surface. The orbital period is 24 hours (identical to Earth).

Weightlessness

The Gravitational Force of the Earth acting on the Satellite provides the Centripetal Force to keep it in orbit. As there is no contact force between astronauts and satellites, they feel apparent weightlessness.

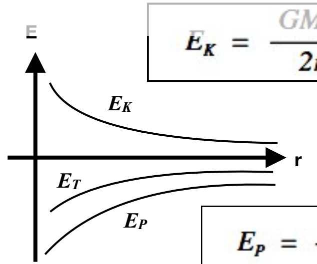
Energy of an Orbital Satellite

$$F_G = F_C$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{\frac{GM}{r}}$$

$$\therefore E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$



$$\boxed{E_K = \frac{GMm}{2r} = -\frac{1}{2}E_P}$$

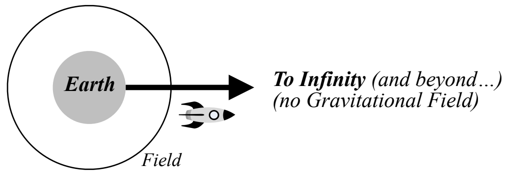
$$\boxed{E_T = E_K + E_P = -\frac{GMm}{2r}}$$

$$\boxed{E_P = -\frac{GMm}{r}}$$

Total energy must be negative, otherwise the satellite would not be in the Gravitational Field.

For Geostationary Orbit, $r = \frac{GM}{v^2} = \frac{GM}{(2\pi r/T)^2} = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$; $T = 86400s$

(C) G-Field: Escaping a Field



If an object, such as a rocket, wishes to escape the influence of a mass' Gravitational Field, it must overcome the Gravitational Potential Energy at the surface of the mass, to reach infinity (where there is no potential energy).

$$\Delta E = E_\infty - E_P$$

$$= \frac{GMm}{R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$\therefore = \sqrt{2gr} \quad \left(as\ g = \frac{GM}{R^2}\right)$$

Most problems involving fields at work should be solved using conservation of energy. However it is to be noted that Total Energy is not the same at all radii. Instead the primary conversion should be the difference in Potential Energy.

For example, a satellite experiencing atmospheric resistance may see a fall in total energy, causing the orbital radius to decrease. While it loses potential energy it gains kinetic energy and gains speed.