3.7 Mathematics: Trigo I & II

This serves as a supplement and is in no way a replacement for your classroom notes. **Overview:**

The study of trigonometry is extremely important. The general intuition is, given a geometric diagram, many times it can be ***defined*** when the given conditions make sure it is possible to draw one and only one such diagram. In such a case, technically, all the side lengths and angles should be able to be determined.

Trigo I deals with the basic concept of trigonometric functions, and it has 3 basic question types: basic manipulation of trigonometric functions, graph drawing, and bearing questions.

Trigo II deals with more advanced concepts, and are usually questions where you have to solve an equation directly or first derive an expression from a diagram. To do this, there are 3 skills involved: the understanding of more advanced trigonometric identities, the ability to list out all solutions from a basic trigonometric equation, as well as the ability to make use of the R-formula. **R-formula means free marks. Please present properly.**

Part 1: Trigo I

<u>Question type 1</u>

Given sin A = $\frac{3}{5}$, cos B = $-\frac{12}{13}$ and A, B belong to the same quadrant, what is the value of

(i) cos (A+B)
(ii) tan (A)
(iii) sin (2B)

We shall look at how to answer this type of questions.

Step 1: Identify the quadrant A and B are in.

sin A is positive while cos B is negative, thus they are in the fourth quadrant.

Step 2: List out sin A, cos A, sin B, cos B.

Therefore, we can see that

$$\sin A = \frac{3}{5} \quad \cos A = -\frac{\sqrt{5^2 - 3^2}}{5} = -\frac{4}{5}$$

$$\cos B = -\frac{12}{13} \quad \sin B = \frac{\sqrt{13^2 - 12^2}}{13} = \frac{5}{13}$$

*Tip: it is advised to remember the basic Pythagorean triples; i.e. (3, 4, 5) (5, 12, 13) are sides of a right angle triangle. These values often appear in examinations.

Step 3: Make use of the *addition/subtraction and double angle formulas to work out what you are required to find out.*

Exercise – Complete the sample question

*Tip: Math is a subject, from the writer's own experience, a subject that cannot be "mugged". The only way to memorise the formulas is to constantly drill yourself with questions until you know when and where to use it. In other words, only with experience will you be able to make use of such formulas.

It is important however to be smart about it. When I first encountered the double angle formulas, I did not think of them as double angle formulas. Instead…

Exercise – derive the double angle formulas based on addition formulas.

Formulas you will need:

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

<u>Question type 2</u>

Graph $y = |2\cos(x - \pi)| + 3$. By drawing a suitable line, find the number of solutions to the equation $|2\cos(x - \pi)| = 3x + 10$ from 0 to π.

I have given the most difficult problem they can ever give of the graphing type. However, the method is extremely standard. Firstly, it is important to understand your basic graphing methods, which are in your math notes. Then, you have to be able to graph out the trigonometric function successfully. Lastly, you will need to be able to identify the line you need to draw, as well as how to draw it.

Finally, after completing your diagram, the **number of intersections** of the two graphs is the number of solutions to your equation.

Exercise – complete the above question

*Tip: Questions of this type can be extremely straightforward if you know what to do. However, on the occasion that you get stuck on drawing the graph SKIP this immediately. Come back when you have a clearer mind, go and do problems that are more brainless than this – **graph drawing requires the most concentration** out of any other types of problems in your math paper.

<u>Question type 3</u>

A ship leaves port A and travels 63km to port B, which is at a bearing of 077° from A. After unloading, it then travels 55km to port C for maintenance, which is at a bearing of 318° from B. How far is port C from port A?

This is a typical bearing question, and the answered asked here is one of the more straightforward kinds. In an actual question, there can be up to 9 parts to the question, each could be enquiring about an **angle, length, angle of elevation/depression, shortest distance and/or (rare instances) area.**

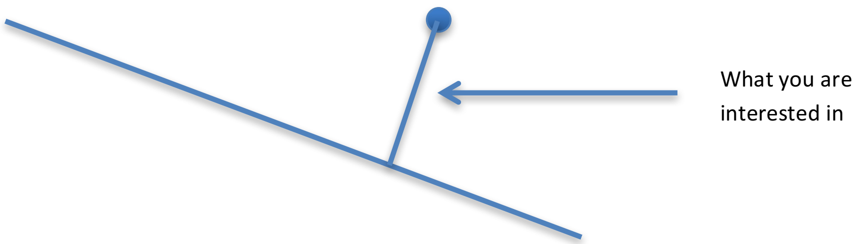
Take note of the following:

1. For angles, it is CRUICIAL that if you cannot derive the angle directly, you have to know the opposite side if this angle is inside a triangle. Meaning, given triangle ABC, if you want to know angle A, it is most important that you know the length of BC.
2. In the same way, for you to know the length BC, you NEED to know the value of angle A. How do you accomplish this? Make use of sine and cosine rule. I know this may be hard for many of you, but do know that **how fast you can do it depends on your experience.** The difference between high scorers and lower scorers in mathematics is how **fast** they

can finish their problems, so they can complete more problems in the paper.

3. Thus, if you are spending too much time, skip it. But make to come back later; these questions worth tons of marks.
4. Don't do nothing. If you don't know whether sine rule works or cosine rule works, TRY BOTH. Math is a subject that given any exam paper and sufficient time to exhaust all methods, you **will** eventually destroy the question.
5. Remember the useful formula for area: it can be expressed as **half the product of the sine of an angle and the two sides that form it. i.e.,** $A = \frac{1}{2}\,bc\sin A$

*Tips: When given the **shortest distance from (point) to (line)** type of questions, always remember that the length you are interested in is the **perpendicular dropped from the POINT to the LINE.** I.e. :



Part 2: Trigo II

Trigo II studies the solving of trigonometric equations. To be honest, the crafting of a **trigonometric expression that represents the length/angle in a geometric diagram** ONLY comes with practice. You should train yourself in listing all solutions given a root solution.

However, there are some tips and tricks to solving trigonometry equations, and this section attempts to allow you to outsmart the exam by giving you a list of methods in which you may **try one by one and exhaust every option**.

1. Reduction to single trigonometric function

This is the most common type of questions and is certainly the most straightforward.

What you basically aim to do is to **factorise** and equate each part to 0.

Let's take a look at a simple example:

$$\sin 2x = \cos x$$

Simple, but enough to illustrate our point.

*Tip: NEVER divide both sides by a variable; if that value can be 0, you just missed out a solution. For this example, what you do is the following:

Notice that you cannot divide cos x over, as you get a function of 2x divided by a function of x, which does not solve your problem. So we expand the sin 2x into a nicer form:

$$\sin 2x = 2 \sin x \cos x = \cos x$$

Now we move ALL terms to one side of the equation:

$$2 \sin x \cos x - \cos x = 0$$

Factorise:

$$\cos x \,(2 \sin x - 1) = 0$$

Thus, we know that either cos x = 0 or the expression in the brackets is 0. We solve for both and arrive at our solution. Completion is left to the reader.

Another recurring theme is what to do when you simply have something like:

$$\frac{2}{3} \sin x = \cos x$$

The solution is pretty simple; divide by cos x on both sides then solve in your calculator for **tangent.** Yes, it might seem simple now, but trust me, understand exam conditions, this type of "simple" equations are the hardest and most annoying to spot.

2. Using identities and manipulate equations, or "prove this identity" question type

It is impossible to put in all of my intuition here, but I'll provide some basic idea on how to approach this type of problems:

Firstly, identify **which side you are working from**. The more complicated the expression, the **more things you can play around with it**. So start with the more complicated side, and try to show it is equal to the less complicated side.

Secondly, do not blindly apply theorems: have a clear goal in mind. Are you trying to factorise out like terms? Are you cancelling something? Do you need a sin squared term – can this can taken from a constant 1?

Lastly, present clearly. Even if you do not arrive at an answer, reaching certain key steps might warrant you some marks. However, if your working is not clear… err, good luck :P

3. Quadratic equation

Many people overlook this possible method, but I have seen problems that can be done with this method before. What you do is to express everything in terms of a single trigonometric function. From here, do a substitution then solve the quadratic equation.

Example:

$$sin^2 x + 2 = 3\cos x$$

We see that it is hard to change cos x, so we try to express everything in terms of cos x.

$$(1 - cos^2 x) + 2 - 3\cos x = 0$$

This implies

$$cos^2 x + 3\cos x - 3 = 0$$

From here, it is possible to key into the calculator or use the quadratic formula to find the value of cos x, from there get the value of x.

Footnote: If you have any queries, especially regarding presentation, please email me at yanggan1998@gmail.com , I'll be more than happy to answer your questions.