*Senior's Tuition And Mentorship Program 3.0 Year 2s*

# 2M.1 Algebra (review)

**Expansion and factorisation**

- Do NOT remember "FOIL". Use **distribution property** of multiplication, which states that for any numbers *a, b, c,* we have $(a+b)\times c = ac + bc$
- Factorisation of quadratic expressions: one must develop sensitivity to quadratic expressions and be able to derive the factorisation within 10 seconds – you would have mastered this technique if you can do this.
- Tips: **FIRST THING – take out any common factors!** E.g. if all the terms are divisible by 2, factorise 2 out first.

  Consider if the middle coefficient if divisible by a factor of a side coefficient: Consider $6x^2 - 5x - 4$. The middle coefficient is not divisible by 2, but both the coefficient of $x^2$ and the constant term are.

  This implies that we must "split" the coefficients such that one product is odd and one is even, i.e. 6 = 2x3 and 4 = 4x1, such that 3x1 – 4x2 = -5. Thus we can factorise the above quadratic expression into $(2x+1)(3x-4)$
- The same idea can be applied to multiples of 3, 5, etc. (well you get the point)

**Factorisation questions: 5-10 seconds per question**

- $4x^2 - 6x + 9$
- $6x^2 + 17x - 14$
- $6x^2 + 7x - 3$
- $-2x^2 - 17x - 21$
- $4x^2 + 5x - 6$
- $11x^2 - 43x - 4$
- $15x^2 - 4x - 4$
- $9x^2 - 3x - 2$
- $3x^2 - 12x + 12$
- $2x^2 - 11x + 12$
- $12x^2 - 28x + 15$
- $2x^2 - 29x + 14$
- $12x^2 - 10x + 2$
- $14x - 15x^2 + 8$
- $3x^2 - 17x + 20$
- $8x^2 + 6x - 9$
- $8x^2 - 42x + 27$
- $2x^2 - 3x - 5$
- $5x^2 + 3x - 2$
- $24x^2 + 42x - 45$

**Must-know common tricks**

These factorisations must be known so well that you can "feel" its presence in questions

- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

- Grouping and *rational roots theorem*. So there are some big expressions that you are supposed to factorise and suppose you are stuck, because you KNOW that you are supposed to use “grouping” but you don't know “how” to group.

  Well, there is this magic called *rational roots theorem* that translates into saying that for any polynomial expression that you see, if it can be factorised into a certain factor $(ax + b)$, then the leading (first) coefficient is divisible by *a* and the constant term is divisible by *b*.

  Sounds simple and obvious enough: say you are supposed to factorise $12x^3 - 16x^2 - 5x + 3$. The divisors of 12 are 1, 2, 3, 4, 6, 12. Divisors of 3 are 1, 3. Now you simply try pairs of these. Try (x+3), (x-3), (x+1), (x-1) all doesn't work. Try (2x+1), it works! Now you factorise into $12x^3 - 16x^2 - 5x + 3 = (2x + 1)(6x^2 - 11x + 3) = (2x + 1)(2x - 3)(3x - 1)$.

**Completing square**

Another fundamental and extremely important technique. To be able to reach the level where one can look at the expression and complete the square directly without writing anything and within 5 seconds, we do the following mental calculations:

1) factorise out the coefficient of $x^2$, this is extremely important!
2) Divide the the coefficient of $x$ by 2, ignoring its sign, then square this value.
3) Subtract this value FROM the constant term, and add this to the end of your square term
4) E.g. $2x^2 - 7x + 1 = 2\left(x^2 - \frac{7}{2}x + \frac{1}{2}\right) = 2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{1}{2}\right] = 2\left(x - \frac{7}{4}\right)^2 - \frac{41}{8}$
5) Remember that all these should be done in your mind. It is not only to speed up your answering, but also train your visualisation of algebraic terms

Universal footnote: Math problem you cannot answer? Concept you don’t understand? Want more practice? You may post your query or request on the Whatsapp group or get an appointment with Yang Gan @ 96605102. I am free the entire Monday afternoon, and you might be able to find me in Raja Block foyer near sandwich machine (that’s my lepak corner).